CHAPTER 1
FUNDAMENTAL CONCEPTS: VECTORS

1.1 (a) $\vec{A}+\vec{B}=(\hat{i}+\hat{j})+(\hat{j}+\hat{k})=\hat{i}+2\hat{j}+\hat{k}$

$$\left|\vec{A}+\vec{B}\right|=(1+4+1)^{\frac{1}{2}}=\sqrt{6}$$

(b) $3\vec{A}-2\vec{B}=3(\hat{i}+\hat{j})-2(\hat{j}+\hat{k})=3\hat{i}+\hat{j}-2\hat{k}$

(c) $\vec{A}\cdot\vec{B}=(1)(0)+(1)(1)+(0)(1)=1$

(d) $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ 0 & 1 & 1\end{vmatrix}=\hat{i}(1-0)+\hat{j}(0-1)+\hat{k}(1-0)=\hat{i}-\hat{j}+\hat{k}$

$$\left|\vec{A}\times\vec{B}\right|=(1+1+1)^{\frac{1}{2}}=\sqrt{3}$$

1.2 (a) $\vec{A}\cdot\left(\vec{B}+\vec{C}\right)=\left(2\hat{i}+\hat{j}\right)\cdot\left(\hat{i}+4\hat{j}+\hat{k}\right)=(2)(1)+(1)(4)+(0)(1)=6$

$$\left(\vec{A}+\vec{B}\right)\cdot\vec{C}=\left(3\hat{i}+\hat{j}+\hat{k}\right)\cdot 4\hat{j}=(3)(0)+(1)(4)+(1)(0)=4$$

(b) $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix}2 & 1 & 0\\ 1 & 0 & 1\\ 0 & 4 & 0\end{vmatrix}=-8$

$$\left(\vec{A}\times\vec{B}\right)\cdot\vec{C}=\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=-8$$

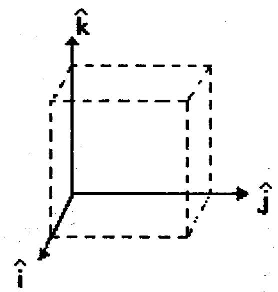
(c) $\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}\cdot\vec{C}\right)\vec{B}-\left(\vec{A}\cdot\vec{B}\right)\vec{C}=4\left(\hat{i}+\hat{k}\right)-2\left(4\hat{j}\right)=4\hat{i}-8\hat{j}+4\hat{k}$

$$\left(\vec{A}\times\vec{B}\right)\times\vec{C}=-\vec{C}\times\left(\vec{A}\times\vec{B}\right)=-\left[\left(\vec{C}\cdot\vec{B}\right)\vec{A}-\left(\vec{C}\cdot\vec{A}\right)\vec{B}\right]$$
$$=-\left[0\left(2\hat{i}+\hat{j}\right)-4\left(\hat{i}+\hat{k}\right)\right]=4\hat{i}+4\hat{k}$$

1.3 $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(a)(a)+(2a)(2a)+(0)(3a)}{\sqrt{5a^2}\sqrt{14a^2}} = \frac{5a^2}{a^2\sqrt{5}\sqrt{14}}$

$$\theta = \cos^{-1}\sqrt{\frac{5}{14}} \approx 53°$$

1.4



(a) $\vec{A} = \hat{i}+\hat{j}+\hat{k}$: *body diagonal*

$A = \left|\vec{A}\cdot\vec{A}\right| = \sqrt{\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{j}+\hat{k}\cdot\hat{k}} = \sqrt{3}$

(b) $\vec{B} = \hat{i}+\hat{j}$: *face diagonal*

$B = \left|\vec{B}\cdot\vec{B}\right| = \sqrt{2}$

(c) $\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 1 & 0\end{vmatrix}$

(d) $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{1-1}{\sqrt{3}\sqrt{2}} = 0 \quad \therefore \theta = 90°$

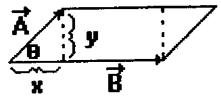
1.6 $\frac{d\vec{A}}{dt} = \hat{i}\frac{d}{dt}(\alpha t)+\hat{j}\frac{d}{dt}(\beta t^2)+\hat{k}\frac{d}{dt}(\gamma t^3) = \hat{i}\alpha+\hat{j}2\beta t+\hat{k}3\gamma t^2$

$$\frac{d^2\vec{A}}{dt^2} = \hat{j}2\beta+\hat{k}6\gamma t$$

1.7 $0 = \vec{A}\cdot\vec{B} = (q)(q)+(3)(-q)+(1)(2) = q^2-3q+2$

$(q-2)(q-1)=0, \qquad q = 1 \text{ or } 2$

1.10



$$y = A\sin\theta$$

$$\mathrm{A} = 2\left(\frac{1}{2}xy\right) + y\left(B - x\right) = xy + yB - xy = AB\sin\theta$$

$$\mathrm{A} = \left|\vec{A}\times\vec{B}\right|$$

1.17 $\vec{v}\left(t\right) = -\hat{i}b\omega\sin\left(\omega t\right) + \hat{j}2b\omega\cos\left(\omega t\right)$

$$\left|\vec{v}\right| = \left(b^2\omega^2\sin^2\omega t + 4b^2\omega^2\cos^2\omega t\right)^{\frac{1}{2}} = b\omega\left(1 + 3\cos^2\omega t\right)^{\frac{1}{2}}$$

$$\vec{a}\left(t\right) = -\hat{i}b\omega^2\cos\omega t - \hat{j}2b\omega^2\sin\omega t$$

$$\left|\vec{a}\right| = b\omega^2\left(1 + 3\sin^2\omega t\right)^{\frac{1}{2}}$$

at $t = 0$, $\left|\vec{v}\right| = 2b\omega$; at $t = \frac{\pi}{2\omega}$, $\left|\vec{v}\right| = b\omega$

1.18 $\vec{v}\left(t\right) = \hat{i}b\omega\cos\omega t - \hat{j}b\omega\sin\omega t + \hat{k}2ct$

$$\vec{a}\left(t\right) = -\hat{i}b\omega^2\sin\omega t - \hat{j}b\omega^2\cos\omega t + \hat{k}2c$$

$$\left|\vec{a}\right| = \left(b^2\omega^4\sin^2\omega t + b^2\omega^4\cos^2\omega t + 4c^2\right)^{\frac{1}{2}} = \left(b^2\omega^4 + 4c^2\right)^{\frac{1}{2}}$$

1.21 $\vec{r}\left(t\right) = \hat{i}\left(1 - e^{-kt}\right) + \hat{j}e^{kt}$

$$\vec{r}\left(t\right) = \hat{i}ke^{-kt} + \hat{j}ke^{kt}$$

$$\vec{r}\left(t\right) = -\hat{i}k^2e^{-kt} + \hat{j}k^2e^{kt}$$