Ph 201

2.1 $F_x = m a_x$

$$a_x = \frac{F_x}{m} \Longrightarrow \frac{dv_x}{dt} = \frac{F_x}{m} \quad (v_x = \dot{x})$$

a) $F_x = F_0 + ct$

$$\ddot{x} = \frac{dv_x}{dt} = \frac{F_0 + ct}{m}$$

$$\dot{x} = \frac{1}{m}\int_0^t F_0 + ct \, dt = \frac{1}{m}\left(F_0 t + \frac{c}{2}t^2\right)$$

$$x = \int_0^t \dot{x}\, dt = \frac{1}{m}\int\left(F_0 t + \frac{c}{2}t^2\right)dt = \frac{1}{m}\left[\frac{F_0}{2}t^2 + \frac{c}{6}t^3\right]$$

b) $F_x = F_0 \sin ct$

$$\dot{x} = \int_0^t \frac{F_0}{m}\sin ct \, dt = +\frac{F_0}{mc}(-\cos ct + 1)$$

$$x = \frac{F_0}{mc}\int_0^t (1 - \cos ct)\,dt = \frac{F_0}{mc}\left(t - \frac{\sin ct}{c}\right)$$

$$x = \frac{F_0}{mc^2}(ct - \sin ct)$$

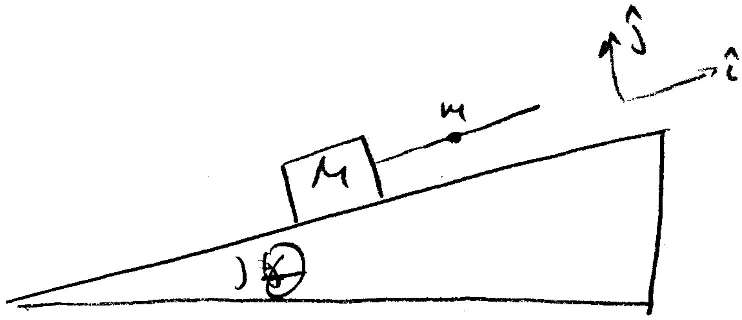
c) $F_x = F_0 e^{ct}$

$$\dot{x} = \frac{F_0}{m}\int_0^t e^{ct}\, dt = \frac{F_0}{mc}\left(e^{ct} - 1\right)$$

$$x = \frac{F_0}{mc}\int_0^t \left(e^{ct} - 1\right) dt = \frac{F_0}{mc}\left(\frac{e^{ct} - 1}{c} - t\right)$$

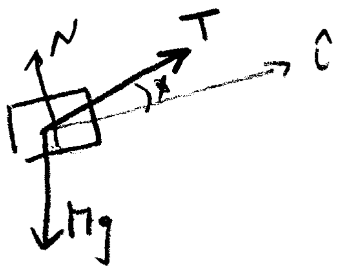
$$= \frac{F_0}{mc^2}\left(e^{ct} - 1 - ct\right) = -\frac{F_0}{mc^2}\left(1 + ct - e^{ct}\right)$$

2.7

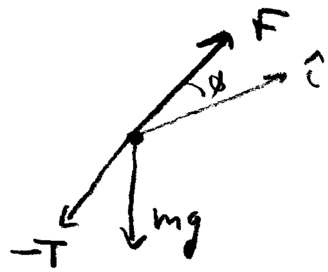


There are <u>two</u> objects in the system, the <u>block</u> and the <u>string</u>.

Forces on the block



Forces on the string



Decompose the forces along the $\hat{\imath}$ + $\hat{\jmath}$ axes

$\hat{\imath}$: $T\cos\phi - Mg\sin\theta = Ma_x$ | $F\cos\phi - T\cos\phi - mg\sin\theta = ma_x$

$\hat{\jmath}$: $T\sin\phi + N - Mg\cos\theta = 0$ | $F\sin\phi - T\sin\phi - mg\cos\theta = 0$

The force exerted by the string on the block is $\bar{T}$. The block exerts an equal and opposite force on the string, $-\bar{T}$. The block is resting on the plane, but the string is not. We will assume the string does not sag, and does not stretch. The string and the block have the <u>same</u> acceleration, $\bar{a}$.

2.7

Solve the $\hat{\imath}$-equations for a_x and set them equal

$$\frac{T}{M}\cos\phi - \cancel{g\sin\theta} = \frac{F}{m}\cos\phi - \frac{T}{m}\cos\phi - g\sin\theta$$

Solve for $T = \left(\frac{mM}{m+M}\right)F$.

Next, plug this into the $\hat{\jmath}$-equation for the string and solve for $\sin\phi$

$$F\sin\phi - \frac{mM}{m+M}F\sin\phi - mg\cos\theta = 0$$

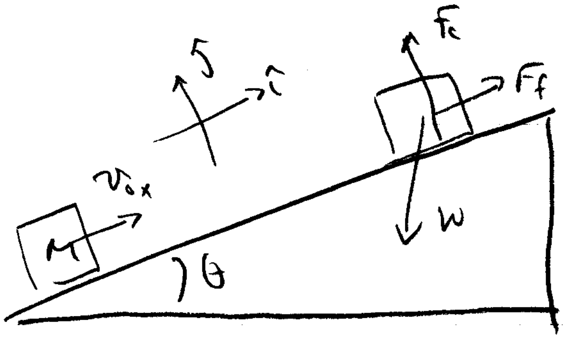
$$F\sin\phi\left(1-\frac{mM}{m+M}\right) = mg\cos\theta$$

$$\sin\phi = \frac{mg\cos\theta}{F\left(1-\frac{mM}{m+M}\right)} = \left(\frac{m^2+mM}{m}\right)\frac{g}{F}\cos\theta .$$

So, we have the magnitude of $\overline{T}$ and the angle it makes with the $\hat{\imath}$-axis (ϕ).

Query: why can't $\phi = 0$?

2-10



$\theta = 30^\circ$
$\mu = 0.1$

The item slides up the plane with constant acceleration, comes to rest and slides back down. We have to break the problem into 2 parts.

up

$$x: \quad W_x + F_{fx} + \cancel{F_{cx}}^{\,0} = M a_x$$
$$-Mg\sin\theta - \mu F_c = M a_x$$

$$y: \quad W_y + \cancel{F_{fy}}^{\,0} + \cancel{F_{cy}}^{\,F_c} = M\cancel{a_y}^{\,0}$$
$$F_c = Mg\cos\theta$$

$$-Mg\sin\theta - \mu Mg\cos\theta = M a_x$$

Since a_x = constant, we have $a_x = -\dfrac{v_{0x}}{t}$.

$$-\cancel{M}g\sin\theta - \mu \cancel{M}g\cos\theta = -\cancel{M}\frac{v_{0x}}{t}$$

Solve for t

$$t = \frac{v_{0x}}{g(\sin\theta + \mu\cos\theta)} = 1.74\,\frac{v_{0x}}{g}$$

To get t down, we need x, the distance slid up the incline:

$$x = \cancel{x_0}^{0} + v_{0x}t + \frac{1}{2}a_x t^2$$

$$x = v_{0x}\,t - \frac{1}{2}\frac{v_{0x}}{t}t^2 =$$

$$x = \left(v_{0x} - \frac{1}{2}v_{0x}\right)t = \frac{v_{0x}}{2}\left(1.74\,\frac{v_{0x}}{g}\right) = 0.852\,\frac{v_{0x}^2}{g}$$

down

$$x: \quad W_x + F_{fx} + \cancel{F_{cx}}^{0} = M a_x$$

$$-Mg\sin\theta + \mu Mg\cos\theta = M a_x$$

$$a_x = g\left(-\sin\theta + \mu\cos\theta\right) = -0.4134\,g$$

Then

$$\cancel{x}^{0} = \cancel{x_0}^{x_{up}} + \cancel{v_{0x}}^{0}t + \frac{1}{2}a_x t^2$$

$$0 = 0.852\,\frac{v_{0x}^2}{g} + \frac{1}{2}\left(-0.4134\,g\right)t^2$$

solve for t

$$t^2 = .852\,\frac{v_{0x}^2}{g}\left(\frac{2}{+0.4134\,g}\right)$$

$$t^2 = 4.122\,\frac{v_{0x}^2}{g^2}$$

$$t = 2.03\,\frac{v_{0x}}{g}$$

The total time is $\quad t = 1.74\,\dfrac{v_{0x}}{g} + 2.03\,\dfrac{v_{0x}}{g}$

$$t = 3.77\,\frac{v_{0x}}{g}$$