

$$2.2 \quad \ddot{x} = \frac{dx}{dt} = v_x \frac{dv_x}{dx} = \dot{x} \frac{d\dot{x}}{dx} \quad (2.3.2 \text{ p } 50)$$

also

$$\ddot{x} = \frac{F_x}{m} \Rightarrow \dot{x} d\dot{x} = \frac{F_x}{m} dx$$

a) $F_x = F_0 + cx$ [$\dot{x} = 0$ when $x=0$ is given]

$$\int_0^{\dot{x}} \dot{x} d\dot{x} = \int_0^x \left(\frac{F_0 + cx}{m} \right) dx$$

$$\frac{\dot{x}^2}{2} = \frac{1}{m} \left(F_0 x + \frac{c}{2} x^2 \right)$$

$$\dot{x} = \sqrt{\frac{2}{m} \left(F_0 x + \frac{c}{2} x^2 \right)}$$

b) $F_x = F_0 e^{-cx}$

$$\frac{\dot{x}^2}{2} = \frac{F_0}{m} \int_0^x e^{-cx} dx$$

$$\dot{x} = \sqrt{\frac{2F_0}{mc} (1 - e^{-cx})}$$

c) $F_x = F_0 \cos cx$

$$\frac{\dot{x}^2}{2} = \frac{F_0}{m} \int_0^x \cos cx dx$$

$$\dot{x} = \sqrt{\frac{2F_0}{ca} (\sin cx)}$$

$$2.3 \quad V(x) = - \int_0^x F_x dx \quad \text{by defn.}$$

$$a) \quad F_x = F_0 + cx$$

$$V(x) = - \int_0^x (F_0 + cx) dx = - \left(F_0 x + \frac{cx^2}{2} \right)$$

$$b) \quad F_x = F_0 e^{-cx}$$

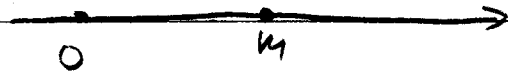
$$\begin{aligned} V(x) &= - \int_0^x e^{-cx} dx = - \frac{F_0}{c} \left(e^{-cx} - e^{-c \cdot 0} \right) \\ &= + \frac{F_0}{c} (e^{-cx} - 1) \end{aligned}$$

$$c) \quad F_x = F_0 \cos cx$$

$$\begin{aligned} V(x) &= - F_0 \int_0^x \cos cx dx = - \frac{F_0}{c} (\sin cx - \sin c \cdot 0) \\ &= - \frac{F_0}{c} \sin cx \end{aligned}$$

Note: I've let the integration constant that appears in each case just be zero.
(i.e. $x_0 = 0$)

2.4



$$F = -kx$$

$$x_0 = 0$$

$$T_0 = \frac{1}{2} k A^2$$

a) $W = -\Delta V(x)$

$$\int_0^x -kx \, dx = -\Delta V(x)$$

$$-\frac{1}{2} k x^2 + 0 = -\Delta V(x) = -V(x) + V(0)$$

$$V(x) = \frac{1}{2} k x^2$$

b) $W = \Delta T$

$$\int_0^x -kx \, dx = \int_{T_0}^T dT$$

$$-\frac{1}{2} k x^2 = T - \frac{1}{2} k A^2$$

$$T(x) = \frac{1}{2} k A^2 - \frac{1}{2} k x^2$$

c) $E(x) = \frac{1}{2} k A^2 - \frac{1}{2} k x^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2 = \text{const.}$

d) $T=0$ when $x = \pm A$.

