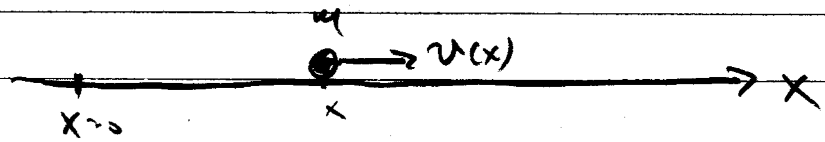
2.6



$v(x) = \frac{\alpha}{x}$ $\quad \alpha > 0$ constant

Start: 2^{nd} law $F_x = m\ddot{x}$. We need $\ddot{x}$

eqn 2.3.2 $\quad \ddot{x} = \dot{x}\frac{d\dot{x}}{dx}$ $\quad$ with $\dot{x} = \frac{\alpha}{x}$

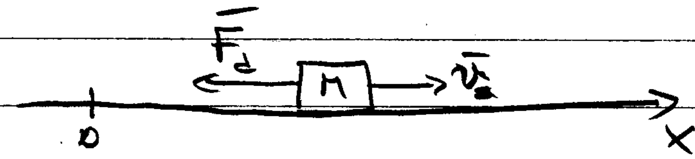
$$\ddot{x} = \frac{\alpha}{x}\frac{d}{dx}\left(\frac{\alpha}{x}\right)$$

$$\ddot{x} = \frac{\alpha}{x}\left(-\frac{\alpha}{x^2}\right)$$

$$\ddot{x} = -\frac{\alpha^2}{x^3}$$

Thus $\quad F_x(x) = -m\frac{\alpha^2}{x^3}$

2.11



$F_{dx} = -cv^{3/2}$ at $x=0$, $v=v_0$

2^{nd} Law $F_d = Ma_x$

we want $\frac{dv}{dt} = v\frac{dv}{dx}$ since F_d is a function of v only (page 51)

$$F = Mv\frac{dv}{dx}$$

$$-cv^{3/2} = Mv\frac{dv}{dx} \qquad \frac{v}{v^{3/2}} = \frac{1}{v^{1/2}} = v^{-1/2}$$

$$-\frac{c}{M}\,dx = v^{-1/2}\,dv$$

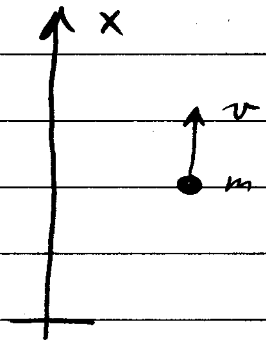
integrate

$$-\frac{c}{M}\int_0^x dx = \int_{v_0}^{0} v^{-1/2}\,dv \qquad \left(\underline{\text{note}}\text{: we want } x \text{ when } v \to 0.\right)$$

$$-\frac{c}{M}x = 0 - 2v_0^{1/2} \qquad \frac{v^{+\frac{1}{2}}}{\frac{1}{2}}$$

$$\underline{x = \frac{2M}{c}v_0^{1/2}}$$

2-12



$$\Sigma F_x = m a_x = m \frac{dv_x}{dt}$$

$$-mg \pm c v_x^2 = m \frac{dv_x}{dt}$$

We have to break the problem into 2 parts: up + down.

up: $$-mg - c v_x^2 = m \frac{dv_x}{dt}$$

We want v_x^2 as a function of x. So use the chain rule to eliminate the dt

$$\frac{dv_x}{dt} = \frac{dv_x}{dx} \cdot \frac{dx}{dt} = v_x \frac{dv_x}{dx}$$

$$-mg - c v_x^2 = m v_x \frac{dv_x}{dx}$$

$$dx = \frac{m v_x \, dv_x}{-mg - c v_x^2}$$

$$(x - 0) = -\int_{v_{0x}}^{v_x} \frac{v_x \, dv_x}{g + \frac{c}{m} v_x^2}$$

look it up: $$\int \frac{x\,dx}{a + bx^2} = \frac{1}{2b} \ln(a + bx^2) \Big]_{\text{bottom}}^{\text{top}}$$

$$x = -\frac{1}{2\frac{c}{m}} \ln\left[\frac{g + \frac{c}{m} v_x^2}{g + \frac{c}{m} v_{0x}^2}\right]$$

Solve for v_x^2

$$-2\frac{m}{c} x = \ln\left[\frac{g + \frac{c}{m} v_x^2}{g + \frac{c}{m} v_{0x}^2}\right]$$

2-12

$$e^{-2\frac{m}{c}x} = \frac{g + \frac{c}{m}v_x^2}{g + \frac{c}{m}v_{0x}^2}$$

$$v_x^2 = \frac{m}{c}\left[\left(g + \frac{c}{m}v_{0x}^2\right)e^{-2\frac{m}{c}x} - g\right]$$

let $k = \frac{c}{m}$ and $A = \frac{m}{c}\left(g + \frac{c}{m}v_{0x}^2\right)$

$$v_x^2 = A e^{-2kx} - \frac{g}{k}$$ period.

Down: $-mg + cv_x^2 = m\frac{dv_x}{dt}$

The steps are the same ... but there is a different sign.

$$x = \int_{v_{0x}}^{v_x} \frac{v_x\, dv_x}{-g + \frac{c}{m}v_x^2} = -\int_{v_{0x}}^{v_x} \frac{v_x\, dv_x}{g - \frac{c}{m}v_x^2}$$

$$x = -\frac{1}{2\left(-\frac{c}{m}\right)} \ln\left[\frac{g - \frac{c}{m}v_x^2}{g - \frac{c}{m}v_{0x}^2}\right]$$

$$2\frac{m}{c}x = \ln\left[\frac{g - \frac{c}{m}v_x^2}{g - \frac{c}{m}v_{0x}^2}\right]$$

$$e^{2\frac{m}{c}x} = \frac{g - \frac{c}{m}v_x^2}{g - \frac{c}{m}v_{0x}^2}$$

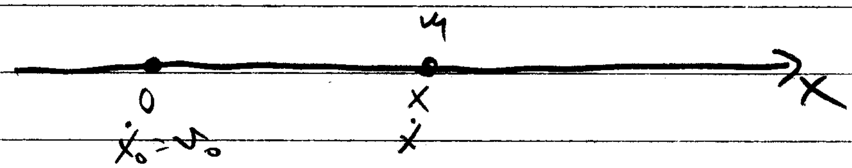
$$v_x^2 = -\frac{m}{c}\left[\left(g - \frac{c}{m}v_{0x}^2\right)e^{2\frac{m}{c}x} - g\right]$$

$$v_x^2 = \frac{g}{k} - B e^{2kx}$$ where $B = \frac{m}{c}\left(g - \frac{c}{m}v_{0x}^2\right)$.

$$\frac{d}{d\dot{x}} e^{-\alpha\dot{x}} = -\alpha e^{-\alpha\dot{x}}$$

2-19

$$\frac{d}{dx}\left(\frac{e^{-\alpha x}}{-\alpha}\right) = e^{-\alpha x}$$



$\dot{x}_0 = v_0 \qquad x_0 = 0$

$F(v) = -Ae^{\alpha v} \qquad A + \alpha = \text{constant}$

$\alpha: \frac{\text{sec}}{m}$

a) 2^{nd} law

$A: \frac{kg\,m}{sec^2}$

$$F(v) = m\ddot{x}$$

$$-Ae^{\alpha\dot{x}} = m\frac{d\dot{x}}{dt}$$

$$\frac{A}{m}\alpha = \frac{\frac{kg\,m}{sec^2}\cdot\frac{sec}{m}}{kg} = \frac{1}{sec}$$

$$\int_0^t dt = \int_{v_0}^{\dot{x}} -\frac{m}{A} e^{-\alpha\dot{x}}\,d\dot{x} = \frac{-m}{-A\alpha}\left[e^{-\alpha\dot{x}}\right]_{v_0}^{\dot{x}}$$

$$t = -\frac{m}{A\alpha}\left[e^{-\alpha\dot{x}} - e^{-\alpha v_0}\right], \text{ Solve for } \dot{x}$$

$$-e^{-\alpha\dot{x}} = \frac{A\alpha}{m}t + e^{-\alpha v_0}$$

$$-\alpha\dot{x} = -\ln\left(\frac{A\alpha}{m}t + e^{-\alpha v_0}\right)$$

$$\dot{x} = +\frac{1}{\alpha}\ln\left(\frac{A\alpha}{m}t + e^{-\alpha v_0}\right)$$

b) Set $\dot{x} = 0$, solve for t

$$1 = \frac{A\alpha}{m}t + e^{-\alpha v_0}$$

$$t = \frac{m}{A\alpha}\left(1 - e^{-\alpha v_0}\right)$$

2-19

c) $\dot{x} = \frac{1}{\alpha} \ln\left(\frac{A\alpha}{m} t + e^{-\alpha v_0} \right)$

$$\frac{dx}{dt} = \frac{1}{\alpha} \ln\left(\frac{A\alpha}{m} t + e^{-\alpha v_0} \right)$$

$$\alpha(x - x_0) = \int_0^t \ln\left(\frac{A\alpha}{m} t + e^{-\alpha v_0} \right) dt$$

This has the form

$$\int \ln(bt+c)\, dt \to \frac{bt+c}{b} \ln(bt+c) - t$$

$$\alpha x = \left[\frac{\frac{A\alpha}{m} t + e^{-\alpha v_0}}{\frac{A\alpha}{m}} \ln\left(\frac{A\alpha}{m} t + e^{-\alpha v_0} \right) - t \right]_0^t$$

$$\frac{\alpha x}{\alpha} = \frac{1}{\alpha}\left(t + \frac{m}{A\alpha} e^{-\alpha v_0} \right) \ln\left(\frac{A\alpha}{m} t + e^{-\alpha v_0} \right) - t$$

put in $t = \frac{m}{A\alpha}\left(1 - e^{-\alpha v_0}\right)$

$$x = \frac{1}{\alpha}\left[\left(\frac{m}{A\alpha} - \cancel{\frac{m}{A\alpha} e^{-\alpha v_0}} + \cancel{\frac{m e^{-\alpha v_0}}{A\alpha}} \right) \cdot \ln\left(\frac{A\alpha}{m} \frac{m}{A\alpha}\left(1 - e^{-\alpha v_0}\right) + e^{-\alpha v_0} \right) - \frac{m}{A\alpha}\left(1 - e^{-\alpha v_0}\right) \right]$$

$$x = \frac{m}{A} \cancelto{0}{\ln 1} - \frac{m}{A\alpha^2}\left(1 - e^{-\alpha v_0}\right)$$

$$x = \frac{m}{A\alpha^2}\left(e^{-\alpha v_0} - 1\right)$$

$\frac{kg}{kg\, m/kg \cdot \left(\frac{m}{sec}\right)^2} = m$