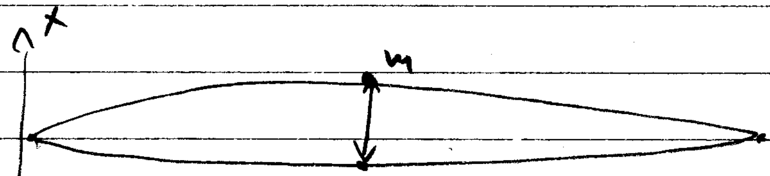
A

3.1



The midpoint of the guitar string oscillates at a frequency of $f_0 = 512\ Hz$. The maximum displacement is $A = 0.002\ m$.

The displacement of the string at time t is

$$X = A \sin \omega_0 t \qquad \text{where} \quad \omega_0 = 2\pi f_0$$

The total energy of ~~a~~ small segment of the string at its midpoint is

$$E = \frac{1}{2} k A^2 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

where the force constant is $k = m\omega_0^2$.

The max. speed occurs when $x = 0$

$$\frac{1}{2} m v_{max}^2 = \frac{1}{2} k A^2$$

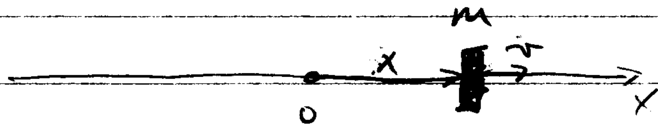
$$\cancel{\frac{1}{2}} \cancel{m} v_{max}^2 = \cancel{\frac{1}{2}} \cancel{m} \omega_0^2 A^2$$

$$v_{max} = \omega_0 A = 2\pi f_0 A = 6.43\ m/sec$$

The max. acceleration is

$$a_{max} = A\omega_0^2 = 2.07 \times 10^4\ m/sec^2$$

3.2



$x = A \sin \omega_0 t \quad , \qquad A = 0.1\,m$

$\omega_0 t = \pi$

$\dot{x} = A\omega_0 \cos\omega_0 t$

when $x = 0$, $\dot{x} = 0.5\,m/sec$. So, when does $x = 0$?
$x = 0$ when $t = 0$ and when $t = \frac{\pi}{\omega_0}, \frac{2\pi}{\omega_0}, \frac{3\pi}{\omega_0}$, etc.
Plug into $\dot{x}$

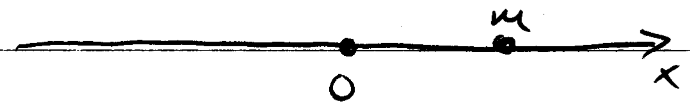
$\dot{x} = A\omega_0 \cos(0) = A\omega_0$

$0.5\,m/sec = 0.1\,m\ \omega_0$, solve for ω_0

$\omega_0 = 5\ sec^{-1}$

The Period is $T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 1.26\ sec$.

3.3



In general $x(t) = A\sin(\omega_0 t + \phi_0)$ or $Ae^{i(\omega_0 t + \phi_0)}$

The initial conditions in this case are

$t=0 \quad x_0 = 0.25\,m \quad \dot{x}_0 = 0.1\ m/sec$

$$x = A\sin(\omega_0 t + \phi_0)$$
$$\dot{x} = A\omega_0\cos(\omega_0 t + \phi_0)$$

We solve for A and ϕ_0, plugging in the initial conditions. $(\omega_0 = 2\pi \cdot 10\,Hz)$

$$x_0 = A\sin\phi_0$$
$$\dot{x}_0 = A\omega_0\cos\phi_0$$

divide the 2 equations

$$\tan\phi_0 = \frac{\omega_0 x_0}{\dot{x}_0} = 157$$

$$\phi_0 = 89.6^\circ = 1.564 \text{ radian}$$

Then solve one of 'em for A

$$A = \frac{x_0}{\sin\phi_0} = \frac{0.25\,m}{.99998} = 0.25\,m$$

Thus $x(t) = 0.25\,m\,\sin(2\pi 10\,Hz\, t + 1.564)$

[Alternative: let $x(t) = A\sin\omega_0 t + B\cos\omega_0 t$; solve for A + B]

3.3 alternate

$$x(t) = A \sin \omega_0 t + B \cos \omega_0 t \quad , \quad \omega_0 = 2\pi(10\,Hz)$$

at $t=0$, $x_0 = 0.25\,m$, $\dot{x}_0 = 0.1\,m/sec$,

Apply the initial conditions

$$0.25\,m = A \cdot 0 + B$$
$$0.1\,m/sec = A\omega_0 \cdot 1 + B\omega_0 \cdot 0$$

$$B = 0.25\,m \quad \text{and} \quad A = \frac{0.1\,m/sec}{\omega_0} = \frac{0.1\,m/sec}{20\pi\,Hz} = 0.0016\,m$$

$$x(t) = 0.0016\,m \sin 20\pi t + 0.25\,m \cos 20\pi t$$

One could have started with

$$x(t) = A \cos \omega_0 t + B \sin \omega_0 t$$

just as well.

3-4 ~~A~~ <u>algebraic</u>

$$X = A\sin(\omega_0 t + \phi_0)$$

$$= A\left[\sin\omega_0 t\cos\phi_0 + \cos\omega_0 t\sin\phi_0\right]$$

$$= (A\cos\phi_0)\cdot\sin\omega_0 t + (A\sin\phi_0)\cos\omega_0 t$$

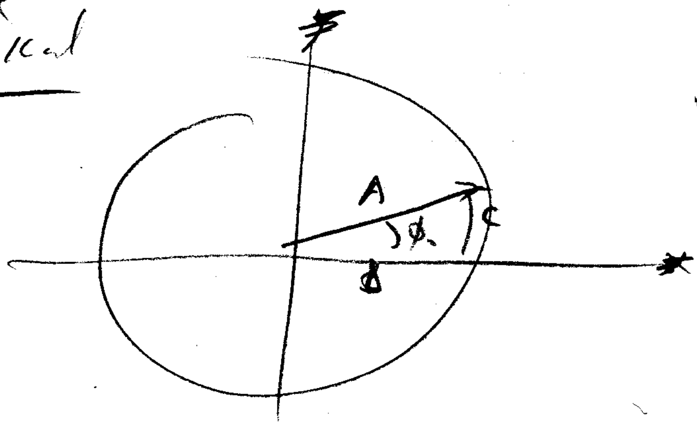
$$X = D\sin\omega_0 t + C\cos\omega_0 t$$

$$\frac{C}{D} = \frac{A\sin\phi_0}{A\cos\phi_0} = \tan\phi_0$$

$$C^2 + D^2 = A^2\sin^2\phi_0 + A^2\cos^2\phi_0$$

$$C^2 + D^2 = A^2$$

<u>graphical</u>



right triangle

$$\sin\phi_0 = \frac{C}{A}$$

$$\cos\phi_0 = \frac{D}{A}$$

etc.

3-5 SHM

$$x = A \sin(\omega_0 t + \phi_0)$$

$$\dot{x} = \omega_0 A \cos(\omega_0 t + \phi_0)$$

When $x = x_1$, then $\dot{x} = \dot{x}_1$;
When $x = x_2$, then $\dot{x} = \dot{x}_2$.

We seek to evaluate ω_0 and A in terms of $\dot{x}_1$, x_1, $\dot{x}_2$ and x_2.
Use the conservation of energy, because the energy E involves x and $\dot{x}$.

$$\frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} k x_2^2$$

Solve for $\frac{k}{m}$: $\frac{k}{m} = \frac{\dot{x}_2^2 - \dot{x}_1^2}{x_1^2 - x_2^2}$

So $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{\dot{x}_2^2 - \dot{x}_1^2}{x_1^2 - x_2^2}}$.

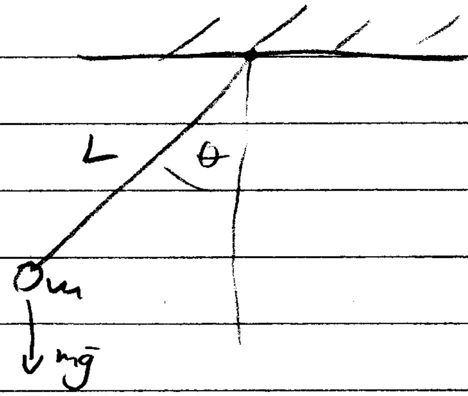
The amplitude is the maximum displacement, $x = A$ at which point $\dot{x} = 0$.

$$\frac{1}{2} k A^2 = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} k x_1^2 \quad \left(= \frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} k x_2^2 \right)$$

Solve for $A^2 = \frac{m}{k} \dot{x}_1^2 + x_1^2$, substitute for $\frac{m}{k}$.

$$A^2 = \frac{\dot{x}_1^2}{\frac{x_1^2 - x_2^2}{\dot{x}_2^2 - \dot{x}_1^2}} + x_1^2 \Rightarrow A = \sqrt{\frac{x_1^2 \dot{x}_2^2 - x_2^2 \dot{x}_1^2}{\dot{x}_2^2 - \dot{x}_1^2}}$$

3.6



$g = \frac{1}{6}\, 9.8\,m/sec^2$
$L = 1m$

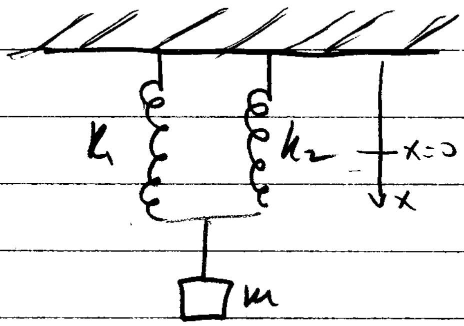
For a simple pendulum, $\omega_0 = \sqrt{\frac{g}{L}}$. The period is $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{L}{g}}$.

$$\frac{1}{2}T = \pi\sqrt{\frac{L}{g}} = \pi\sqrt{\frac{6}{9.8}\,sec^2}$$

$$\frac{1}{2}T = 2.46\ sec$$

3.7

Parallel



The net force on the mass m is

$$(-k_1x)+(-k_2x) = m\ddot{x}$$

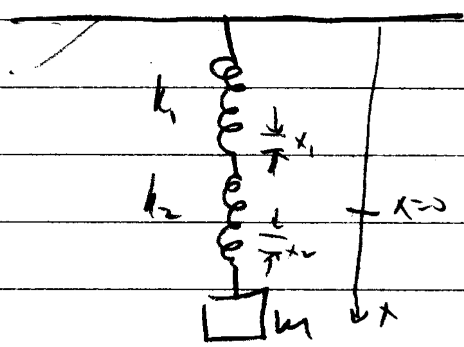
the two springs are stretched the same amount.

Factor the common x

$$(-k_1+k_2)x = m\ddot{x}$$

This is the equation of motion for a restoring force with constant $k = k_1+k_2$. So the natural frequency is $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_1+k_2}{m}}$

Series



In this case the displacement of the mass is $x = x_1+x_2$. The 2 springs are not stretched the same amount, but each exerts the same force.

If the mass and springs are in equilibrium, then $F = -k_1x_1 = -k_2x_2 = -k\,x = m\ddot{x}$

We have $x_1 = -\frac{F}{k_1}$ and $x_2 = -\frac{F_{\text{total}}}{k_2}$ and $x = x_1+x_2$

$$m\ddot{x} = -k\left(\frac{m\ddot{x}}{-k_1}+\frac{m\ddot{x}}{-k_2}\right) \Rightarrow k\left(\frac{1}{k_1}+\frac{1}{k_2}\right)=1$$

So $k = \frac{k_1k_2}{k_1+k_2}$ and $\omega_0^2 = \frac{k_1k_2}{(k_1+k_2)m}$.