4.1 $\bar{F} = -\nabla V$

a) $\bar{F} = -\nabla(cxyz + C)$

$= -\left(\hat{x}\frac{\partial}{\partial x}(cxyz + C) + \hat{y}\frac{\partial}{\partial y}(cxyz + C) + \hat{z}(cxyz + C)\right)$

$= -\left(\hat{x}\ cyz + \hat{y}\ cxz + \hat{z}\ cxy\right)$

b) $\bar{F} = -\nabla(\alpha x^2 + \beta y^2 + \gamma z^2 + C)$

$= -(\hat{x}\ 2\alpha x + \hat{y}\ 2\beta y + \hat{z}\ 2\gamma z)$

c) $\bar{F} = -\nabla\left(c e^{-(\alpha x + \beta y + \gamma z)}\right)$

$= \left[\hat{x}(+\alpha) + \hat{y}(+\beta) + \hat{z}(+\gamma)\right] c e^{-(\alpha x + \beta y + \gamma z)}$

d) $\bar{F} = -\nabla(cr^n)$

$= -\hat{r}\frac{\partial}{\partial r} cr^n$

$= -\hat{r}\ ncr^{n-1}$ (text uses $\hat{r} = e_r$)

4.2 $\nabla \times \bar{F} = 0$?

a) $\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$

$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-0) = 0$

b) $\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & z^2 \end{vmatrix}$

$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(-1-1) \neq 0$

c) $\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & z^2 \end{vmatrix}$

$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(1-1) = 0$

d) $\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = 0$

F_x eqn 1.12.89, b, c

$\hat{r} \cdot \hat{i} = \sin\theta \cos\phi$

$\hat{r} \cdot \hat{j} = \sin\theta \sin\phi$

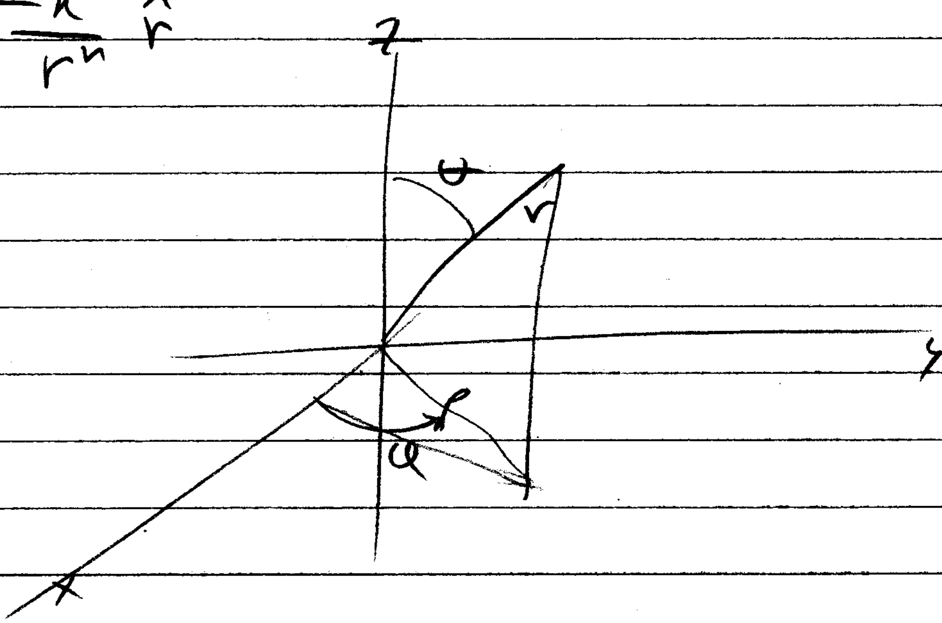
$\hat{r} \cdot \hat{k} = \cos\theta$

$\sin\theta = \frac{\sqrt{x^2+y^2}}{r}$ $\quad \sin\phi = \frac{y}{\sqrt{x^2+y^2}}$

$\cos\theta = \frac{z}{r}$ $\quad \cos\phi = \frac{x}{\sqrt{x^2+y^2}}$

4.2 cont. Spherical to cartesian components

$$\bar{F} = \frac{-k}{r^n}\hat{r}$$



$$\sin\theta = \frac{\rho}{r}$$

$$\cos\theta = \frac{z}{r}$$

$$\cos\phi = \frac{x}{\rho}$$

$$\sin\phi = \frac{y}{\rho}$$

$$\rho = \sqrt{x^2+y^2}$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\bar{F} = \left(\hat{\imath}\,\frac{\rho}{r}\frac{x}{\rho} + \hat{\jmath}\,\frac{\rho}{r}\frac{y}{\rho} + \hat{k}\,\frac{z}{r}\right)\left(\frac{-k}{\cancel{x^2+y^2+z}\,r^n}\right)$$

$$\bar{F} = \frac{-k}{r^{n+1}}\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$$

$$= -\frac{k}{\left(\sqrt{x^2+y^2+z^2}\right)^{n+1}}\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$$

Evidently $\nabla \times \bar{F} = 0$.

for example $\frac{\partial}{\partial x}F_y = \frac{\partial}{\partial y}F_x = 0$, etc.

4.3

a) $\nabla \times \bar{F} = 0$?

$$\nabla \times \bar{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & cx^2 & z^3 \end{vmatrix}$$

$$= \hat{\imath}(0-0) - \hat{\jmath}(0-0) + \hat{k}(2cx - x)$$

So we require $(2c-1)x = 0 \Rightarrow c = \frac{1}{2}$

b) $$\nabla \times \bar{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{z}{y} & \frac{cxz}{y^2} & \frac{x}{y} \end{vmatrix}$$

$$= \hat{\imath}\left(-\frac{x}{y^2} - \frac{cx}{y^2}\right) - \hat{\jmath}\left(\frac{1}{y} - \frac{1}{y}\right) + \hat{k}\left(\frac{cz}{y^2} + \frac{z}{y^2}\right)$$

For both $\hat{\imath}$ + $\hat{k}$ components to be zero, we must have $c = -1$.