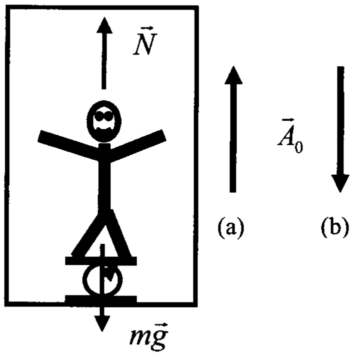
CHAPTER 5
NONINERTIAL REFERENCE SYSTEMS

5.1 (a) The non-inertial observer believes that he is in equilibrium and that the net force acting on him is zero. The scale exerts an upward force, $\vec{N}$, whose value is equal to the scale reading --- the "weight," W', of the observer in the accelerated frame. Thus



$$\vec{N} + m\vec{g} - m\vec{A}_0 = 0$$

$$N - mg - mA_0 = N - mg - m\frac{g}{4} = N - \frac{5}{4}mg = 0$$

$$W' = N = \frac{5}{4}mg = \frac{5}{4}W$$

$$W' = 150 lb.$$

(b) The acceleration is downward, in the same direction as $\vec{g}$

$$N - mg + m\left(\frac{g}{4}\right) = 0 \quad W' = W - \frac{W}{4} = \frac{3}{4}W$$

$$W' = 90 lb.$$

5.2 (a) $\vec{F}_{cent} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r}')$

For $\vec{\omega} \perp \vec{r}'$, $\vec{F}_{cent} = m\omega^2 r' \hat{e}_{r'}$

$$\omega = 500\,s^{-1} = 1000\pi\,s^{-1}$$

$$\vec{F}_{cent} = 10^{-6}\times(1000\pi)^2\times 5\,\hat{e}_r = 5\pi^2 \text{ dynes outward}$$

(b) $$\frac{F_{cent}}{F_g} = \frac{m\omega^2 r'}{mg} = \frac{(1000\pi)^2 5}{980} = 5.04\times 10^4$$

5.3 $m\vec{g} + \vec{T} - m\vec{A}_\circ = 0$ (See Figure 5.1.2)

$$-mg\,\hat{j} + T\cos\theta\,\hat{j} + T\sin\theta\,\hat{i} - m\left(\frac{g}{10}\right)\hat{i} = 0$$

$T\cos\theta = mg$, and $T\sin\theta = \dfrac{mg}{10}$

$$\tan\theta = \frac{1}{10}, \qquad \theta = 5.71^\circ$$

$$T = \frac{mg}{\cos\theta} = 1.005mg$$

5.4 The non-inertial observer thinks that $\vec{g}'$ points downward in the direction of the hanging plumb bob… Thus

$$\vec{g}' = \vec{g} - \vec{A}_\circ = g\,\hat{j} - \frac{g}{10}\hat{i}$$

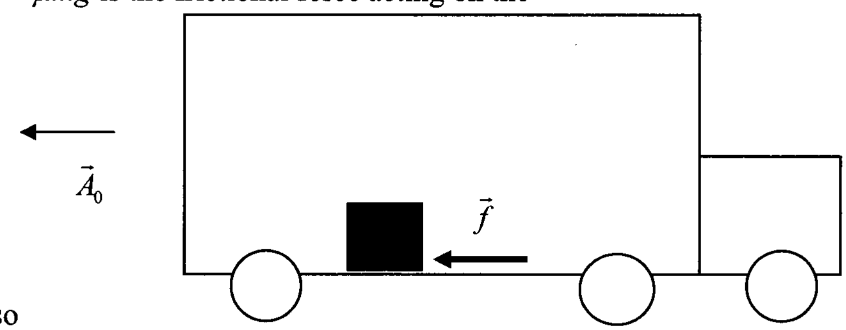
For small oscillations of a simple pendulum:

$$T = 2\pi\sqrt{\frac{1}{g'}}$$

$$g' = \sqrt{g^2 + \left(\frac{g}{10}\right)^2} = 1.005g$$

$$T = 2\pi\sqrt{\frac{1}{1.005g}} = 1.995\pi\sqrt{\frac{1}{g}}$$

5.5 (a) $f = -\mu mg$ is the frictional force acting on the



box, so

$\vec{f} - m\vec{A}_0 = m\vec{a}'$ ($\vec{a}'$ is the acceleration of the box relative to the truck. See Equation 5.1.4b.) Now, $\vec{f}$ the only real force acting horizontally, so the acceration relative to the road is

(b) $a = \frac{f}{m} = -\frac{\mu mg}{m} = -\mu g = -\frac{g}{3}$

(For + in the direction of the moving truck, the – indicates that friction opposes the forward sliding of the box.)

$A_\circ = -\frac{g}{2}$ (The truck is decelerating.)

from above, $ma - mA_0 = ma'$ so

(a) $a' = a - A_\circ = -\frac{g}{3} + \frac{g}{2} = \frac{g}{6}$

5.6 (a) $\vec{r} = \hat{i}\left(x_\circ + R\cos\Omega t\right) + \hat{j}R\sin\Omega t$

$\dot{\vec{r}} = -\hat{i}\Omega R\sin\Omega t + \hat{j}\Omega R\cos\Omega t$

$\dot{\vec{r}}\cdot\dot{\vec{r}} = v^2 = \Omega^2 R^2$ $\quad\therefore\ \underline{v = \Omega R}$ circular motion of radius R

(b) $\dot{\vec{r}}' = \dot{\vec{r}} - \vec{\omega}\times\vec{r}'$ where $\vec{r}' = \hat{i}x' + \hat{j}y'$

$= -\hat{i}\Omega R\sin\Omega t + \hat{j}\Omega R\cos\Omega t - \omega\hat{k}\times\left(\hat{i}x' + \hat{j}y'\right)$

Where $\vec{a}$ is the acceleration of the asteroid in the x, y frame of reference, $\vec{A}, \vec{A}_\varepsilon$ are the accelerations of the asteroid and the Earth in the fixed, inertial frame of reference.

1^{st}: examine: $\vec{A}-\vec{A}_\varepsilon-\vec{\Omega}\times\vec{\Omega}\times\vec{r}$

$$=\vec{\omega}\times\vec{\omega}\times\vec{R}-\vec{\Omega}\times\vec{\Omega}\times\vec{R}_\varepsilon-\vec{\Omega}\times\vec{\Omega}\times\left(\vec{R}-\vec{R}_\varepsilon\right)$$

$$=\left(\vec{\omega}\times\vec{\omega}-\vec{\Omega}\times\vec{\Omega}\right)\times\vec{R}=-\left(\omega^2-\Omega^2\right)\vec{R} \qquad \text{note: } \vec{\omega}=\omega\hat{k},\ \vec{\Omega}=\Omega\hat{k}$$

Thus:

$$\vec{a}=\left(\Omega^2-\omega^2\right)\vec{R}-2\vec{\Omega}\times\vec{v}$$

Therefore:

$$\left(\hat{i}\ddot{x}+\hat{j}\ddot{y}\right)=\left(\Omega^2-\omega^2\right)\left[\hat{i}R\cos\left(\Omega-\omega\right)t-\hat{j}R\sin\left(\Omega-\omega\right)t\right]-2\hat{j}\Omega\dot{x}+2\hat{i}\Omega\dot{y}$$

Thus:

$$\ddot{x}=\left(\Omega^2-\omega^2\right)R\cos\left(\Omega-\omega\right)t+2\Omega\dot{y}$$

$$\ddot{y}=-\left(\Omega^2-\omega^2\right)R\sin\left(\Omega-\omega\right)t-2\Omega\dot{x}$$

Let $\ddot{x}=\left(\Omega-\omega\right)\dot{y}$ and $\ddot{y}=\left(\Omega-\omega\right)\dot{x}$

Then, we have

$$\dot{y}=\left(\Omega-\omega\right)R\cos\left(\Omega-\omega\right)t+\frac{2\Omega}{\left(\Omega-\omega\right)}\dot{y} \qquad \text{which reduces to}$$

$$\dot{y}=-\left(\Omega-\omega\right)R\cos\left(\Omega-\omega\right)t$$

Integrating … $\boxed{y=-R\sin\left(\Omega-\omega\right)t}\rightarrow 0$ at $t=0$

Also,

$$-\dot{x}\left(\Omega-\omega\right)=-\left(\Omega^2-\omega^2\right)R\sin\left(\Omega-\omega\right)t-2\Omega\dot{x}$$

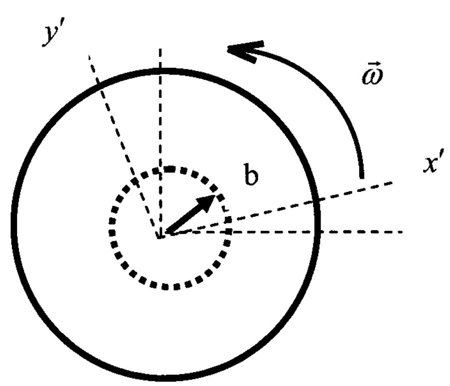
or $\dot{x}=\left(\Omega+\omega\right)R\sin\left(\Omega-\omega\right)t+2\Omega\dot{x}$

$$\dot{x}=-\left(\Omega-\omega\right)R\sin\left(\Omega-\omega\right)t$$

Integrating … $x=R\cos\left(\Omega-\omega\right)t+const$

$\boxed{x=R\cos\left(\Omega-\omega\right)t-R_\varepsilon}\rightarrow R-R_\varepsilon$ at $t=0$

5.8 Relative to a reference frame fixed to the turntable the cockroach travels at a constant speed v' in a circle. Thus



$$\vec{a}'=-\frac{v'^2}{b}\hat{e}_{r'}.$$

Since the center of the turntable is fixed.

$\vec{A}_\circ=0$

The angular velocity, ω, of the turntable is constant, so

$$\vec{\omega} = \omega \hat{k}', \text{ with } \dot{\vec{\omega}} = 0$$

$$\vec{r}' = b\hat{e}_{r'}, \text{ so } \vec{\omega}\times(\vec{\omega}\times\vec{r}') = -b\omega^2\hat{e}_{r'}$$

$$\vec{v}' = v'\hat{e}_{\theta'}, \text{ so } \vec{\omega}\times\vec{v}' = -\omega v'\hat{e}_{r'}$$

From eqn 5.2.14, $\vec{a} = \vec{a}' + 2\vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}')$ and putting in terms from above

$$a_{r'} = -\frac{v'^2}{b} - 2\omega v' - b\omega^2$$

For no slipping $|\vec{F}| \le \mu_s mg$, so $|\vec{a}| \le \mu_s g$

$$\frac{v'^2}{b} + 2\omega v' + b\omega^2 \le \mu_s g$$

$$v_m'^2 + 2\omega b v_m' + b^2\omega^2 - b\mu_s g = 0$$

$$v_m' = -\omega b \pm \sqrt{\omega^2 b^2 - b^2\omega^2 + b\mu_s g}$$

Since v' was defined positive, the +square root is used.

$$v_m' = -\omega b + \sqrt{b\mu_s g}$$

(b)

$$\vec{v}' = -v'\hat{e}_{\theta'}$$

$$\vec{\omega}\times\vec{v}' = +\omega v'\hat{e}_{r'}$$

$$a_{r'} = -\frac{v'^2}{b} + 2\omega v' - b\omega^2$$

$$\frac{v'^2}{b} - 2\omega v' + b\omega^2 \le \mu_s g$$

$$v_m' = \omega b + \sqrt{b\mu_s g}$$

5.9 As in Example 5.2.2, $\vec{\omega} = \frac{V_\circ}{\rho}\hat{j}'$ and $\vec{A}_\circ = \frac{V_\circ^2}{\rho}\hat{i}'$

For the point at the front of the wheel:

$$\ddot{\vec{r}}' = \frac{V_\circ^2}{b}\hat{j}' \text{ and } \vec{v}' = -V_\circ\hat{k}'$$

$$\dot{\vec{\omega}} = 0$$

$$\vec{\omega}\times\vec{r}' = \frac{V_\circ}{\rho}\hat{k}'\times(-b\hat{j}') = \frac{V_\circ b}{\rho}\hat{i}'$$

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}') = \frac{V_\circ}{\rho}\hat{k}'\times\frac{V_\circ b}{\rho}\hat{i}' = \frac{V_\circ^2 b}{\rho}\hat{j}'$$

$$\vec{\omega}\times\vec{v}' = \frac{V_\circ}{\rho}\hat{k}'\times(-V_\circ\hat{k}') = 0$$

$$\vec{a} = \ddot{\vec{r}}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}') + \vec{A}_\circ = \frac{V_\circ^2}{\rho}\hat{i}' + \left(\frac{V_\circ^2}{b} + \frac{V_\circ^2 b}{\rho^2}\right)\hat{j}'$$

$$= +\frac{g}{l}x\sin\omega' t - \frac{g}{l}y\cos\omega' t - 2\omega'\dot{x}'$$

Collecting terms and neglecting terms in ω'^2:

$$\left(\ddot{x} + \frac{g}{l}x\right)\cos\omega' t + \left(\ddot{y} + \frac{g}{l}y\right)\sin\omega' t = 0$$

$$\left(\ddot{x} + \frac{g}{l}x\right)\sin\omega' t - \left(\ddot{y} + \frac{g}{l}y\right)\cos\omega' t = 0$$

5.21 $T = \dfrac{24}{\sin\lambda}$ hours

$$T = \frac{24}{\sin 19^\circ} = 73.7 \text{ hours}$$

5.22 Choose a coordinate system with the origin at the center of the wheel, the x' and y' axes pointing toward fixed points on the rim of the wheel, and the z' axis pointing toward the center of curvature of the track. Take the initial position of the x' axis to be horizontal in the $-\vec{V}_\circ$ direction, so the initial position of the y' axis is vertical.

The bicycle wheel is rotating with angular velocity $\dfrac{V_\circ}{b}$ about its axis, so …

$$\vec{\omega}_1 = \hat{k}'\frac{V_\circ}{b}$$

A unit vector in the vertical direction is:

$$\hat{n} = \hat{i}'\sin\frac{V_\circ t}{b} + \hat{j}'\cos\frac{V_\circ t}{b}$$

At the instant a point on the rim of the wheel reaches its highest point:

$$\vec{r}' = b\hat{n} = b\left(\hat{i}'\sin\frac{V_\circ t}{b} + \hat{j}'\cos\frac{V_\circ t}{b}\right)$$

Since the coordinate system is moving with the wheel, every point on the rim is fixed in that coordinate system.

$$\dot{\vec{r}}' = 0 \quad \text{and} \quad \ddot{\vec{r}}' = 0$$

The $x'y'z'$ coordinate system also rotates as the bicycle wheel completes a circle around the track:

$$\vec{\omega}_2 = \hat{n}\frac{V_\circ}{\rho} = \frac{V_\circ}{\rho}\left(\hat{i}'\sin\frac{V_\circ t}{b} + \hat{j}'\cos\frac{V_\circ t}{b}\right)$$

The total rotation of the coordinate axes is represented by:

$$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 = \frac{V_\circ}{\rho}\left(\hat{i}'\sin\frac{V_\circ t}{b} + \hat{j}'\cos\frac{V_\circ t}{b}\right) + \hat{k}'\frac{V_\circ}{b}$$

$$\dot{\vec{\omega}} = \frac{V_\circ^2}{\rho b}\left(\hat{i}'\cos\frac{V_\circ t}{b} - \hat{j}'\sin\frac{V_\circ t}{b}\right)$$