4.6 $\sin 58° = x/(5.0 \text{ m})$, $x = 4.2$ m.

4.7 The statue is 16 m from the point of incidence, and since the ray-triangles are similar, 4 m : 16 m as 3 m : Y and $Y = 12$ m.

4.8 At the first mirror, $\theta_r = \theta_i$. For the second, $\theta_i' = 90 - \theta_r = 90 - \theta_i$ and $\theta_r' = \theta_i'$, so $\theta_r' = 90 - \theta_i$.

4.9 $n_i \sin\theta_i = n_t \sin\theta_t$, $\sin 30° = 1.52 \sin\theta_t$, $\theta_t = \sin^{-1}(1/3.04)$, so $\theta_t = 19°13'$.

4.10
$$P_{\text{transverse}} = mv_i \sin\theta_i$$
$$= mv_i \sin\theta_t$$
where "m" is the presumed mass. But $v_i = \frac{s_0}{t}, v_f = \frac{BP}{t}$. So

$$(s_0)\sin\theta_i = (BP)\sin\theta_t$$
$$\sin\theta_i = \frac{BP}{s_0}\sin\theta_t$$

The factor $\frac{BP}{s_0}$ corresponds to r_{ti}.

4.11 The slope of the curve is $n_{it} = n_i/n_t$. Slope $\sim 0.75/1.00$, so that $n_t \simeq 1.33$. This suggests that the dense medium is water.

4.12 $\theta_t = \sin^{-1}[(\sin 45°)/2.42] = 17°$, the angular deviation is $45° - 17° = 28°$.

4.13 $\theta_t = \sin^{-1}[(n_w/n_g)\sin\theta_i] = \sin^{-1}[(8/9)\sin 45°] = 39°$. For a ray incident in the glass at this angle,
$\theta_t = \sin^{-1}[(n_g/n_w)\sin 39°] = \sin^{-1}[(9/8)\sin 39°] = 45°$.

4.14 (a) $n_{ti} = n_t/n_i = (c/v_t)/(c/v_i) = v_i/v_t = \nu\lambda_i/\nu\lambda_t = \lambda_i/\lambda_t$. Therefore $\lambda_t = \lambda_i 3/4 = 9$ cm. (b) $\sin\theta_i = n_{ti}\sin\theta_t$, $\theta_t = \sin^{-1}[(3/4)\sin 45°] = 32°$.

4.15 $\lambda_t = \lambda_i/n_{ti} = 600/1.5 = 400$ nm, violet light.

4.16 $1.00 \sin 55° = n \sin 40°$; $n = 1.27$ or 1.3.

4.17 $1.33 \sin 35° = 1.00 \sin\theta_t$; $\theta_t = 50°$.

4.18 For $\theta_i = 0$, 10, 20, 30, 40, 50, 60, 70, 80,90 degrees, $\theta_t = 0$, 6.7, 13.3, 19.6, 25.2, 30.7, 35.1, 38.6, 40.6, 41.8 degrees respectively.

4.19 Consider one ray on each side of the beam, with a perpendicular separation D. The width of the beam on the interface is $D\cos\theta_i$. Likewise, the width of the beam at the interface is $D'\cos\theta_t$, where D' is the perpendicular separation (width) of the rays in the glass, and $D\cos\theta_i = D'\cos\theta_t$. (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so

$$\begin{aligned}\cos\theta_t &= (1 - \sin^2\theta_t)^{1/2} \\ &= (1 - \sin^2\theta_i/n_g^2)^{1/2}\end{aligned}$$

so

$$D' = \frac{D\cos\theta_i}{\left(1 - \frac{\sin^2\theta_i}{n_g^2}\right)^{1/2}}$$

4.20 (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so $\sin(60.0°) = n_t \sin G_\epsilon$. Diameter of emerging beam (D) is related to the difference in horizontal displacement of red and violet light (h) by $D\cos(60.0°) = h$ (See Problem 4.19). Red: $\sin\theta_{\text{red}} = \sin(60.0°)/n_{\text{red}} = (\sqrt{3}/2)/(1.505), \theta_{\text{red}} = 35.1°$; $\tan\theta_{\text{red}} = h_{\text{red}}/10.0$ cm so $h_{\text{red}} = (10.0\text{ cm})\tan(35.1°) = 7.04$ cm. Violet: $\sin\theta_{\text{violet}} = \sin(60.0°)/n_{\text{violet}} = (\sqrt{3}/2)/(1.545)$; $\theta_{\text{violet}} = 34.1°$; $h_{\text{violet}} = (10.0\text{ cm})\tan(34.1°) = 6.77$ cm. $D = h/\cos(60.0°) = (h_{\text{red}} - h_{\text{violet}})/\cos(60.0°) = (7.04 - 6.77)/(0.5) = 0.54$ cm.

4.21 $n_a/n_w = d_A/d_R = 1/1.333 = 0.750 = 3/4$.

4.22 Using Figure S.4.17, $1.00\sin 35° = 1.50\sin\theta_{t1}$; $\theta_{t1} = 22.48°$ and $\cos 22.48° = (2.00\text{ cm})/L$; $L = 2.16$ cm or 2.2 cm.

4.23 $\sin\theta_i = n\sin\theta_i/2$; since $\sin 2\alpha = 2\sin\alpha\cos\alpha$, $\sin\theta_i = 2\sin(\theta_i/2)\cos(\theta_i/2)$ and so setting these two expressions equal we get

$$1.70\sin(\theta_i/2) = 2\sin(\theta_i/2)\cos(\theta_i/2); \quad \cos\theta_i/2 = 0.85;$$

$$31.79° = \theta_i/2; \quad \theta_i = 63.6°.$$

4.24 The glass will change the depth of the object from d_R to d_A, where $d_A/d_R = 1.00/1.55$; but $d_R = 1.00$ mm; hence, $d_A = 0.645$ mm and the camera must be raised 1.00 mm − 0.645 mm = 0.355 mm.

4.25 $d_{A1}/d_{R1} = 1.50/1.33$; $d_{R1} = 1.00$ m; $d_{A1} = 1.1278$ m; $d_{R2} = d_{A1} + 0.02$ m; $d_{A2}/d_{R2} = 1.00/1.50$; $d_{A2} = 1.3278(1.00/1.50) = 0.885$ m.

4.26 The number of waves per unit length along $\overline{AC}$ on the interface equals $(\overline{BC}/\lambda_i)/(\overline{BC}\sin\theta_i) = (\overline{AD}/\lambda_t)/(\overline{AD}\sin\theta_t)$. Snell's law follows on multiplying both sides by c/ν.

4.27 With the origin in the plane of incidence, $z = 0$; with the origin on the interface $y = 0$ so $(\vec{k}_i \cdot \vec{r}) \rightarrow k_{ix}x$

$$(\vec{k}_r \cdot \vec{r} + \epsilon_r) \rightarrow k_{rx}x + \epsilon_r$$

$$(\vec{k}_t \cdot \vec{t} + \epsilon_t) \rightarrow k_{tx}x + \epsilon_t$$

and as $\epsilon_r = \epsilon_t = 0$, Eq. (4.19) becomes $k_{ix} = k_{rx} = k_{tx}$ or $k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t$. Since $k = 2\pi/\lambda$

$$\frac{\sin\theta_i}{\lambda_i} = \frac{\sin\theta_t}{\lambda_t},$$

which is the condition derived in Problem (4.26) for wave front continuity.

4.28 Let τ be the time for the wave to move along a ray from b_1 to b_2, from a_1 to a_2, and from a_1 to a_3. Thus $\overline{a_1a_2} = \overline{b_1b_2} = v_i\tau$ and $\overline{a_1a_3} = v_i\tau$.
$\sin\theta_i = \overline{b_1b_2}/\overline{a_1b_2} = v_i/\overline{a_1b_2}$, $\sin\theta_t = \overline{a_1a_3}/\overline{a_1b_2} = v_t/\overline{a_1b_2}$,
$\sin\theta_r = \overline{a_1a_2}/\overline{a_1b_2} = v_i/\overline{a_1b_2}$, $\sin\theta_i/\sin\theta_t = v_i/v_t = n_t/n_i = n_{ti}$ and $\theta_i = \theta_r$.

4.29 $n_i \sin\theta_i = n_t \sin\theta_t$, $n_i(\hat{k}_i \times \hat{u}_n) = n_t(\hat{k}_t \times \hat{u}_n)$, where $\hat{k}_i$, $\hat{k}_t$ are unit propagation vectors. Thus $n_t(\hat{k}_t \times \hat{u}_n) - n_i(\hat{k}_i \times \hat{u}_n) = 0$, $(n_t\hat{k}_t - n_i\hat{k}_i) \times \hat{u}_n = 0$. Let $n_t\hat{k}_t - n_i\hat{k}_i = \vec{\Gamma} = \Gamma\hat{u}_n$. Γ is often referred to as the *astigmatic constant*; Γ is the difference between the projections of $n_t\hat{k}_t$ and $n_i\hat{k}_i$ on $\hat{u}_n$; in other words, take the dot product $\vec{\Gamma} \cdot \hat{u}_n$:
$\Gamma = n_t \cos\theta_t - n_i \cos\theta_i$.