$$\theta_1 = \varphi + \alpha$$

$$\sin\theta_1 = \sin(\varphi + \alpha)$$

$$= \sin\varphi\cos\alpha$$

$$+ \cos\varphi\sin\alpha \simeq \sin\varphi + \sin\alpha = h/R + h/s.$$

(4.4) $n_1 \sin\theta_1 = n_2 \sin\theta_2$; $n_1(h/R + h/s_o) = n_2(h/R - h/s_i)$,
$n_i/s_o + n_2/s_i = (n_2 - n_1)/R$.

5.6 From Snell's Law $n_1\theta_i = n_2\theta_t$; $\tan\theta_i = y_o/s_o$ and $\tan\theta_t = -y_i/s_i$ since y_i is negative; thus $\theta_i = y_o/s_o$ and $\theta_t = -y_i/s_i$, therefore
$M_T = y_i/y_o = -(n_1 s_i)/(n_2 s_o)$.

5.7 From Eq. (5.8), $1/30.0 + 1.33/s_i = (1.333 - 1.000)/5.0$; $s_i = 40.7$ cm and $M_T = -1.02$, thus the image is 3.05 cm tall.

5.8 First surface: $n_1/s_o + n_2/s_i = (n_2 - n_1)/R$, $1/1.2 + 1.5/s_i = 0.5/0.1$, $s_i = 0.36$ m (real image 0.36 m to the right of first vertex). Second surface $s_o = 0.20 - 0.36 = -0.16$ m (virtual object distance).
$1.5/(-0.16) + 1/s_i = -0.5/(-0.1)$, $s_i = 0.069$ m. The final image is real ($s_i > 0$), inverted ($M_T < 0$), and 6.9 cm to the right of the second vertex.

5.9 At the first surface from Eq. (5.8), $1/30.0 + 1.33/s_i = (1.333 - 1.000)/5.0$ and $s_i = 40.7$; a real image right of the vertex. For the second surface $s_o = -30.7$ cm and the image will be right of the second vertex, so $1.33/(-30.7) + 1/s_i = (1.000 - 1.333)/(-5.000)$; and $s_i = 9.09$ cm to the right of the second surface. The first surface produces a magnification of $M_T = -1.02$, thus the intermediate image is 3.05 cm tall. The second surface produces a magnification of

$$M_T = -(n_1 s_i)/(n_2 s_o) = -(1.333)(9.09)/(1.000)(-30.7) = 0.395$$

and the total magnification is the product of the two, viz., -0.403. The image is real, inverted and minified.

5.10 (5.16) $1/f = (n-1)(1/R_1 - 1/R_2)$ where $R_2 = -R_1$, so

$$1/f = (n-1)(2/R_1)$$

$$R_1 = (n-1)(2)(f) = (1.5-1)(2)(+10.0 \text{ cm})$$

$$= 10.0 \text{ cm}$$

(5.17) $1/s_o + 1/s_i = 1/f$; $s_o = 1.0$ cm;
$1/s_i = 1/f - 1/s_o = 1/10.0 \text{ cm} - 1/1.0 \text{ cm} = -9.0/10.0$; $s_i = -1.1$ cm.
(5.25) $M_T = -s_i/s_o = -(-1.1 \text{ cm})/(1.0 \text{ cm}) = +1.1$. Image is virtual, erect, and larger than the object.

5.11 (5.14) In the thin lens limit ($d \to 0$) becomes
$n_m/s_o + n_m/s_i = (n_\ell - n_m)(1/R_1 - 1/R_2)$ so,
$1/s_o + 1/s_i = 1/f = (n_\ell - n_m/n_m)(1/R_1 - 1/R_2)$. For a double concave lens $R_1 < 0$, $R_2 > 0$, so that $(1/R_1 - 1/R_2) < 0$. For air lens in water, $n_\ell < n_m$, so that $n_\ell - n_m < 0$; $1/f > 0$, lens is converging.

5.12 (5.15) $1/s_o + 1/s_i = (n_\ell - 1)(1/R_1 - 1/R_2)$ so,

$$1/s_i = (n_\ell - 1)(1/R_1 - 1/R_2) - 1/s_o; \quad 1/s_i = -13.3 \text{ cm}.$$

(5.25) $M_T = -s_i/s_o = -13.3/20.0 = +0.67$. Image is virtual, erect, and smaller than the object.

5.13 $1/8 + 1.5/s_i = 0.5/(-20)$. At the first surface, $s_i = -10$ cm. Virtual image 10 cm to the left of first vertex. At second surface, object is *real* 15 cm from second vertex. $1.5/15 + 1/s_i = -0.5/10$, $s_i = -20/3 = -6.66$ cm. Virtual, to left of second vertex.

5.14 (a) (5.17) $1/s_o + 1/s_i = 1/f$ so,
$1/s_i = 1/f - 1/s_o = 1/(5.00 \text{ cm}) - 1/(1000 \text{ cm})$; $s_i = 5.03 \text{ cm} = 50.3$ mm.
(b) (5.25) $M_T = -s_i/s_o = -5.03 \text{ cm}/1000 \text{ cm} = -.00503$.
Image size $= |M_T|(\text{object size}) = (.00503)(1700 \text{ mm}) = 8.55$ mm.

5.15 $s_o + s_i = s_o s_i/f$ to minimize $s_o + s_i$, $(d/ds_o)(s_o + s_i) = 0 = 1 + ds_i/ds_o$ or

$$\frac{d}{ds_o}\left(\frac{s_o s_i}{f}\right) = \frac{s_i}{f} + \frac{s_o}{f}\frac{ds_i}{ds_o} = 0$$

5.24 (a) From the Gaussian lens equation $1/15.0 + 1/s_i = 1/3.00$, $s_i = +3.75$ m. (b) Computing the magnification, we obtain $M_T = -s_i/s_o = -3.75/15.0 = -0.25$. Because the image distance is positive, the image is *real.* Because the magnification is negative, the image is *inverted,* and because the absolute value of the magnification is less than one, the image is *minified.* (c) From the definition of magnification, it follows that $y_i = M_T y_o = (-0.25)(2.25 \text{ m}) = -0.563$ m, where the minus sign reflects the fact that the image is inverted. (d) Again from the Gaussian equation $1/17.5 + 1/s_i = 1/3.00$ and $s_i = +3.62$ m. The entire equine image is only 0.13 m long.

5.25 (5.17) $1/f = 1/s_o + 1/s_i$ so,

$$1/s_i = 1/f - 1/s_o = 1/(-30) - 1/(+10) = -4/3.$$

$s_i = -7.5$ cm. (5.25) $M_T = -s_i/s_o = -(-7.5)/30 = 1/4 = 0.25$.
(Image size) $= M_T$(object size) $= (0.25)(6.00 \text{ cm}) = 1.50$ cm.

The Image is virtual, 7.5 cm in front of the lens, erect, and 1.50 cm tall.

5.26 $|R_1| = |R_2|$, so (5.16) becomes

$$1/f = (n_\ell - 1)(1/R_1 - 1/(-R_1)) = (n_\ell - 1)(2/R_1) = 1/s_i + 1/s_o;$$

$s_o + s_i = 60$ cm (Image real). $|M_T| = (25 \text{ cm})/(5.0 \text{ cm}) = 5 = s_i/s_o$ so,

$$s_i = 5(s_o); \quad s_o + 5(s_o) = 60 \text{ cm}.$$

$$s_o = 10 \text{ cm}; \quad s_i = 50 \text{ cm}.$$

$$1/f = 1/s_o + 1/s_i = 1/10 + 1/50 = 6/50; \quad f = 8.3 \text{ cm}.$$

$$R_1 = (n_\ell - 1)(2)(f) = (1.5 - 1)(2)(8.3 \text{ cm}) = 8.3 \text{ cm}.$$

5.27 $1/s_o + 1/s_i = 1/f$ and $M_T = -s_i/s_o = -1/2$ hence $1/s_o + 2/s_o = 1/f$ but $s_o = 60.0$ cm, hence $f = 20.0$ cm; draw a ray cone from an axial image point, it enters the edges of the lens and focuses at 30.0 cm and then spreads out beyond to create a blur on the screen; from the geometry entrance pupil, bends at L_1 so it just misses the edge of the A.S., and then bends at L_2 so as to pass by the edge of the exit pupil.

5.48 Figures P.5.48a and P.5.48b.

5.49 No—although she might be looking at you.

5.50 The mirror is parallel to the plane of the painting, and so the girl's image should be directly behind her and not off to the side.

5.51 $1/s_o + 1/s_i = -2/R$. Let $R \to \infty$: $1/s_o + 1/s_i = 0$, $s_o = -s_i$, and $M_T = +1$. Image is virtual, same size, and erect.

5.52 From Eq. (5.50), $1/100 + 1/s_i = -2/80$, and so $s_i = -28.5$ cm. Virtual ($s_i < 0$), erect ($M_T > 0$), and minified. (Check with Table 5.5.)

5.53 (5.48) $1/s_o + 1/s_i = -2/R$, $R = 0.5$ ft.

$$1/s_i = -2/R - 1/s_o = -2/(0.5 \text{ ft}) - 1/(-5 \text{ ft}), \quad s_i = -5/21 = -0.24 \text{ ft}.$$

$$M_T = -s_i/s_o = (-0.24)/(5) = 0.048.$$

Image is virtual (seen in the mirror), erect, and 0.048 times the object size.

5.54 Ant has 3 images: from lens, from mirror, back out from lens.
(i) (5.17) $1/f = 1/s_o + 1/s_i$ so,
$1/s_i = 1/f - 1/s_o = 1/50.0 - 1/250 = 4/250$, $s_i = 62.6$ cm (between lens and mirror). (ii) (5.48) $1/s_o + 1/s_i = -2/R$ so,
$1/s_i = -2/R - 1/s_o = 1/s_o$, ($R = \infty$), $s_i = -187.5$ cm (virtual image).
(iii) $1/s_i = 1/f - 1/50 = 1/50.0 - 1/(250 + 187.5)$, $s_i = +56.5$ cm. Real image, (left of lens).

5.55 Image on screen must be real, therefore s_i is positive.
$1/25 + 1/100 = -2/R$, $5/100 = -2/R$, $R = -40$ cm.

5.56 The image is erect and minified. That implies (Table 5.5) a convex spherical mirror.

5.57 From Eq. (5.8), $1/\infty + n/s_i = (n-1)/R$; $s_i = 2R$; $n/2R = (n-1)/R$; $n = 2$.

5.66 See Table 5.3. For $f < s_o < 2f$, a real inverted image is made with $\infty > s_i > 2f$. If this image is directed back at the same angles, the final image will occur at the original object. So, for either type of mirror, it should be placed at the image of the lens (at s_i).

5.67 $M_T = -s_i/s_o$, so, $s_i = -M_T s_o = -1.5s_o$.
(5.50) $1/f = 1/s_o + 1/s_i = 1/s_o + 1/(-1.5s_o)$; $1/10 = 1/3s_o$;
$s_o = 10/3 = 3.3$ cm.
Note in Table 5.5, $s_o < f$ for an erect, magnified image.

5.68 $f = -R/2 = 30$ cm, $1/20 + 1/s_i = 1/30$, $1/s_i = 1/30 - 1/20$. $s_i = -60$ cm, $M_T = -s_i/s_o = 60/20 = 3$. Image is virtual ($s_i < 0$), erect ($M_T > 0$), located 60 cm behind mirror, and 9 inches tall.

5.69 Treat the first surface as a mirror with radius of curvature R.
(5.49) $f_m = -R/2$, which is where the parallel reflected rays converge.
Lens: (5.16) $1/f_\ell = (n_\ell - 1)(1/R_1 - 1/R_2)$; $R_1 = -R$, $R_2 = +R$ so
$1/f_\ell = (2-1)(1/(-R) - 1/R) = -2/R$; $f_\ell = -R/2 = f_m$.

5.70 Image is rotated through 180°.

5.71 From Eq. (5.65),

$$NA = (2.624 - 2.310)^{1/2} = 0.550, \quad \theta_{\max} = \sin^{-1} 0.550 = 33°22'.$$

Maximum acceptance angle is $2\theta_{\max} = 66°44'$. A ray at 45° would quickly leak out of the fiber; in other words, very little energy fails to escape, even at the first reflection.

5.72 Considering Eq. (5.66), $\log 0.5 = -0.30 = -\alpha L/10$, and so $L = 15$ km.

5.73 From Eq. (5.65), $NA = 0.232$ and $N_m = 9.2 \times 10^2$.

5.74 (5.68) $\Delta t = (Ln_f/c)(n_f/n_c - 1)$, so $\Delta t/L = (n_f/c)(n_f/n_c - 1)$.
$\Delta t/L = 1.500/(3 \times 10^{-4}$ km/ns$)(1.500/1.485 - 1) = 50.51$ km/ns.

5.75 $M_T = -f/x_o = -1/x_o\mathcal{D}$. For the human eye $\mathcal{D} \approx 58.6$ diopters.
$x_o = 230{,}000 \times 1.61 = 371 \times 10^3$ km,

$$\mathcal{A} = \begin{bmatrix} 1 & -\mathcal{D}_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d/n_l & 1 \end{bmatrix} \begin{bmatrix} 1 & -\mathcal{D}_1 \\ 0 & 1 \end{bmatrix}$$

$$\mathcal{A} = \begin{bmatrix} 1 & 0.05 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -0.1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.02 & -0.052 \\ 0.4 & 0.96 \end{bmatrix}$$

$$(1.02)(0.96) - (0.4)(-0.052) = 0.979 + 0.0208 = 1.$$

6.17 We have

$$\det \mathcal{A} = a_{11}a_{22} - a_{12}a_{21} = 1 - (D_1 + D_2)d_{21}/n_{t1} + D_1 D_2 d_{21}^2/n_{t1}^2$$
$$+ (D_1 + D_2)d_{21}/n_{t1} + D_1 D_2 d_{21}/n_{t1} = 1.$$

6.18 $h_1 = n_{i1}(1 - a_{11})/(-a_{12}) = (D_2 d_{21}/n_{t1})f = -(n_{t1} - 1)d_{21}f/R_2 n_{t1}$, from Eq. (5.64) where $n_{t1} = n_l$; $h_2 = n_{t2}(a_{22} - 1)/(-a_{12}) = -(D_1 d_{21}/n_{t1})f$ from Eq. (5.70), $h_2 = -(n_{i1} - 1)d_{21}f/R_1 n_{t1}$.

6.19 $\mathcal{A} = \mathcal{R}_2 \mathcal{F}_{21} \mathcal{R}_1$, but for the planar surface

$$\mathcal{R}_2 = \begin{bmatrix} 1 & \mathcal{D}_2 \\ 0 & 1 \end{bmatrix} \text{ and } \mathcal{D}_2 = (n_{t1} - 1)/(-R_2) \text{ but } R_2 = \infty \mathcal{R}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which is the unit matrix, hence $\mathcal{A} = \mathcal{F}_{21}\mathcal{R}_1$.

6.20 $\mathcal{D}_1 = (1.5-1)/0.5 = 1$ and $\mathcal{D}_2 = (1.5-1)/-(-0.25) = 2 \mathcal{A} = \begin{bmatrix} 0.6 & -2.6 \\ 0.2 & 0.8 \end{bmatrix}$ and $|\mathcal{A}| = 0.48 + 0.52 = 1$.

6.21 From the equation above (6.34),

$$-0.2 = -a_{21} = (n_{t1} - 1)\left\{\frac{1}{R_1} + \frac{1}{R_2}\left[\frac{(n_{t1} - d_{21})}{R_1 n_{t1}} - 1\right]\right\}.$$

Solving for the reciprocal of the second radius gives

$$\frac{1}{R_2} = -\left[a_{21} + \frac{(n_{t1} - 1)}{R_1}\right] \frac{R_1 n_{t1}}{(n_{t1} - 1)} \frac{1}{(n_{t1} - d_{21} - R_1 n_{t1})} = 4 \text{ cm}^{-1}.$$

Then $R_2 = 0.25$ cm.

6.22 $\mathcal{R}_1 = \begin{bmatrix} 1 & -\mathcal{D}_1 \\ 0 & 1 \end{bmatrix}$ from (6.16) where

$$\mathcal{D}_1 = (n-1)/R_1 = (3/2-1)/-10.0 \text{ cm} = -0.050 \text{ cm}^{-1}.$$

$$\mathcal{T}_{21} = \begin{bmatrix} 1 & 0 \\ d_{21}/n & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1.00/1.50 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.67 & 1 \end{bmatrix}$$

$$\mathcal{R}_2 = \begin{bmatrix} 1 & -\mathcal{D}_2 \\ 0 & 1 \end{bmatrix}$$

but $R_2 = \infty$, so $\mathcal{D}_2 = 0$.

(6.29) $A = \mathcal{R}_2\mathcal{T}_{21}\mathcal{R}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.67 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0.05 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.05 \\ 0.67 & 1.03 \end{bmatrix}$

Check: $|A| = 1(1.03) - (0.05)(0.67) = 1$.

$$\begin{bmatrix} n_t\alpha_t \\ y_t \end{bmatrix} = A \begin{bmatrix} n_i\alpha_i \\ y_i \end{bmatrix}$$

$$\alpha_t = 0, \quad n_i = 1, \quad y_t = y_i.$$

$$\begin{bmatrix} 0 \\ y_i \end{bmatrix} = A \begin{bmatrix} \alpha_i \\ y_i \end{bmatrix} = \begin{bmatrix} 1 & 0.05 \\ 0.67 & 1.03 \end{bmatrix}\begin{bmatrix} \alpha_i \\ y_i \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ y_i \end{bmatrix} = \begin{bmatrix} \alpha_i + (0.05)y_i \\ (0.67)\alpha_i + (1.03)y_i \end{bmatrix}$$

$$0 = \alpha_i + (0.05)y_i, \quad y_i = (0.67)\alpha_i + (1.03)y_i,$$

both yield $\alpha_i = (-0.05)(2.0) = -0.10$ or 0.10 radians above the axis.

6.23 (6.34) $1/f = -a_{12} = -(\mathcal{D}_1 + \mathcal{D}_2 - \mathcal{D}_1\mathcal{D}_2 d/n_\ell)$;

$\mathcal{D}_1 = (n_\ell - 1)/R_1 = (1.5-1)/0.5 = 1.0$;

$\mathcal{D}_2 = (n_\ell - 1)/R_2 = (1.5-1)/(-0.25) = -2.0$.

$1/f = -(1.0 - 2.0 - (10)(2.0)(0.3)/1.5, \quad f = 0.71.$

$\overline{V_1H_1} = (1)(1 - a_{11})/-a_{12}$,

(6.34) $a_{11} = 1 - \mathcal{D}_2 d/n_\ell = 1 - (-2.0)(0.3)/1.5 = 1.4$.

$\overline{V_1H_1} = (1-1.4)/1.4 = -0.29$.