Chapter 2 Solutions

2.1 $(0.003)(2.54 \times 10^{-2}/580 \times 10^{-9})$ = number of waves = 131, $c = \nu\lambda$, $\lambda = c/\nu = 3 \times 10^8/10^{10}$, $\lambda = 3$ cm. Waves extend 3.9 m.

2.2 $\lambda = c/\nu = 3 \times 10^8/5 \times 10^{14} = 6 \times 10^{-7}$ m $= 0.6\mu$ m.
$\lambda = 3 \times 10^8/60 = 5 \times 10^6$ m $= 5 \times 10^3$ km.

2.3 $v = \lambda\nu = 5 \times 10^{-7} \times 6 \times 10^8 = 300$ m/s.

2.4 The time between the crests is the period, so $\tau = 1/2$ s; hence $\nu = 1/\tau = 2.0$ Hz. As for the speed $v = L/t = 4.5$ m/1.5 s $= 3.0$ m/s. We now know τ, ν, and v and must determine λ. Thus, $\lambda = v/\nu = 3.0$ m/s/2.0 Hz $= 1.5$ m.

2.5 $v = \nu\lambda = 3.5 \times 10^3$ m/s $= \nu(4.3$ m$)$; $\nu = 0.81$ kHz.

2.6 $v = \nu\lambda = 1498$ m/s $= (440$ Hz$)\lambda$; $\lambda = 3.40$ m.

2.7 $v = (10$ m$)/2.0$ s$) = 5.0$ m/s; $\nu = v/\lambda = (5.0$ m/s$)/(0.50$ m$) = 10$ Hz.

2.8 $v = \nu\lambda = (\omega/2\pi)\lambda$ and so $\omega = (2\pi/\lambda)v$.

2.9

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$
$\sin\theta$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$
$\cos\theta$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$
$\sin(\theta-\pi/4)$	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin(\theta-\pi/2)$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$
$\sin(\theta-3\pi/4)$	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0
$\sin(\theta+\pi/2)$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$

θ	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin\theta$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0
$\cos\theta$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin(\theta-\pi/4)$	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$
$\sin(\theta-\pi/2)$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1
$\sin(\theta-3\pi/4)$	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$
$\sin(\theta+\pi/2)$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1

$\sin\theta$ leads $\sin(\theta-\pi/2)$.

2.10

x	$-\lambda/2$	$-\lambda/4$	0	$\lambda/4$	$\lambda/2$	$3\lambda/4$	λ
$\kappa x = 2\pi/\lambda x$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\cos(\kappa x-\pi/2)$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$
$\cos(\kappa x+3\pi/4)$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$

2.11

t	$-\tau/2$	$-\tau/4$	0	$\tau/4$	$\tau/2$	$3\tau/4$	τ
$\omega t = 2\pi/\tau$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	π
$\sin(\omega t+\pi/4)$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$
$\sin(\pi/4-\omega t)$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$

2.12 Comparing y with Eq. (2.13) tells us that $A = 0.02$ m. Moreover, $2\pi/\lambda = 157\ \text{m}^{-1}$ and so $\lambda = 2\pi/(157\text{m}^{-1}) = 0.0400$ m. The relationship between frequency and wavelength is $v = \nu\lambda$, and so $\nu = v/\lambda = 1.2\ \text{m/s}/0.0400\ \text{m} = 30$ Hz. The period is the inverse of the frequency, and therefore $\tau = 1/\nu = 0.033$ s.

2.13 (a) $\lambda = (4.0 - 0.0)\ \text{m} = 4.0$ m. (b) $v = \nu\lambda$, so $\nu = v/\lambda = (20.0\ \text{m/s})/(4.0\ \text{m}) = 5.0$ Hz. (c) Eq. (2.28) $\psi(x,t) = A\sin(kx - \omega t + \epsilon)$. From the figure, $A = 0.020$ m; $k = 2\pi/\lambda = 2\pi/(4.0\ \text{m}) = 0.5\pi\ \text{m}^{-1}$; $w = 2\pi\nu = 2\pi(5.0\ \text{Hz}) = 10.0\pi$ rad/s

At $t = 0$, $x = 0$, $\psi(0,0) = -0.020$ m;
$\psi(0,0) = (0.020\ \text{m})\sin(0.5\pi(0) - 10.0\pi(0) + \epsilon) = (0.020\ \text{m})\sin(\epsilon)$;
$\sin(\epsilon) = -1$; $\epsilon = -\pi/2$. $\psi(x,t) = (0.020\ \text{m})\sin(0.5\pi x - 10.0\pi t - \pi/2)$

2.14 (a) $\lambda = (30.0 - 0.0)\ \text{cm} = 30.0$ cm. (c) $v = \nu\lambda$, so $\nu = v/\lambda = (100\ \text{cm/s})/(30.0\ \text{cm}) = 3.33$ Hz

2.15 (a) $\tau = (0.20 - 0.00)$ s $= 0.20$ s. (b) $\nu = 1/\tau = 1/(0.20s) = 5.00$ Hz. (c) $v = \nu\lambda$, so $\lambda = v/\nu = (40.0 \text{ cm/s})/(5.00 \text{ s}^{-1}) = 8.00$ cm.

2.16 $\psi = A\sin 2\pi(\kappa x - \nu t)$, $\psi_1 = 4\sin 2\pi(0.2x - 3t)$. (a) $\nu = 3$, (b) $\lambda = 1/0.2$, (c) $\tau = 1/3$, (d) $A = 4$, (e) $v = 15$, (f) positive x
$\psi = A\sin(kx + \omega t)$, $\psi_2 = (1/2.5)\sin(7x + 3.5t)$. (a) $\nu = 3.5/2\pi$, (b) $\lambda = 2\pi/7$, (c) $\tau = 2\pi/3.5$, (d) $A = 1/2.5$, (e) $v = 1/2$, (f) negative x

2.17 Form of Eq. (2.26) $\psi(x,t) = A\sin(kx - \omega t)$ (a) $w = 2\pi\nu$, so $\nu = \omega/2\pi = (20.0 \text{ rad/s})/2\pi$, (b) $k = 2\pi/\lambda$, so $\lambda = 2\pi/k = 2\pi/(6.28 \text{ rad/m}) = 1.00$ m, (c) $\nu = 1/\tau$, so $\tau = 1/\nu = 1/(10.0/\pi \text{ Hz}) = 0.10\pi s$, (d) From the form of ψ, $A = 30.0$ cm, (e) $v = \omega/k = (20.0 \text{ rad/s})/(6.28 \text{ rad/m}) = 3.18$ m/s, (f) Negative sign indicates motion in $+x$ direction.

2.18 $\partial^2\psi/\partial x^2 = -k^2\psi$ and $\partial^2\psi/\partial t^2 = -k^2v^2\psi$. Therefore $\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = (-k^2 + k^2)\psi = 0$.

2.19 $\partial^2\psi/\partial x^2 = -k^2\psi$; $\partial^2\psi/\partial t^2 = -w^2\psi$; $w^2/v^2 = (2\pi\nu)^2/v^2 = (2\pi/\lambda)^2 = k^2$; therefore, $\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = (-k^2 + k^2)\psi = 0$

2.20 $\psi(x,t) = A\cos(hx - \omega t - (\pi/2)) =$
$A\{\cos(kx - \omega t)\cos(-\pi/2) - \sin(kx - \omega t)\sin(-\pi/2)\} = A\sin(kx - \omega t)$

2.21 $v_y = -\omega A\cos(kx - \omega t + \epsilon)$, $a_y = -\omega^2 y$. Simple harmonic motion since $a_y \propto y$.

2.22 $\tau = 2.2 \times 10^{-15}$ s; therefore $\nu = 1/\tau = 4.5 \times 10^{14}$ Hz; $v = \nu\lambda$, $\lambda = v/\nu = 6.6 \times 10^{-7}$ m and $k = 2\pi/\lambda = 9.5 \times 10^6 \text{ m}^{-1}$.
$\psi(x,t) = (10^3 V/m)\cos[9.5 \times 10^6 m^{-1}(x + 3 \times 10^8 (m/s)t)]$. It's cosine because $\cos 0 = 1$.

2.23 $y(x,t) = C/[2 + (x + vt)^2]$.

Chapter 3 Solutions

3.1 Compare $E_y = 2\cos[2\pi \times 10^{14}(t - x/c) + \pi/2]$ to $E_y = A\cos[2\pi\nu(t - x/v) + \pi/2]$. (a) $\nu = 10^{14}$ Hz, $v = c$, and $\lambda = c/\nu = 3 \times 10^8/10^{14} = 3 \times 10^{-6}$ m, moves in positive x-direction, $A = 2$ V/m, $\epsilon = \pi/2$ linearly polarized in the y-direction. (b) $B_x = 0$, $B_y = 0$, $B_z = E_y/c$.

3.2 $E_z = 0$, $E_y = E_x = E_0 \sin(kz - \omega t)$ or cosine; $B_z = 0$, $B_y = -B_x = E_y/c$, or if you like,

$$\vec{E} = \frac{E_0}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})\sin(kz - \omega t), \qquad \vec{B} = \frac{E_0}{c\sqrt{2}}(\hat{\jmath} - \hat{\imath})\sin(kz - \omega t).$$

3.3 First, by the right-hand rule, the directions of the vectors are right. Then $kE = \omega B$ and so $(2\pi/\lambda)E = \omega B = 2\pi\nu B$, hence $E = \lambda\nu B = c$.

3.4 $\partial E/\partial x = -kE_0 \sin(kx - \omega t)$; $-\partial B/\partial t = -\omega B_0 \sin(kx - \omega t)$; $-kE_0 = -\omega B_0$; $E_0 = (\omega/k)B_0$ and Eq. (2.33) $\omega/k = c$.

3.5 (a) The electric field oscillates along the line specified by the vector $-\hat{\imath} + \sqrt{3}\hat{\jmath}$. (b) To find E_0, dot $\vec{E}_0$ with itself and take the square root, thus $E_0 = \sqrt{9 + 27}10^4$V/m $= 6 \times 10^4$ V/m. (c) From the exponential $\vec{k} \cdot \vec{r} = (\sqrt{5}x + 2y)(\pi/3) \times 10^7$, hence $\vec{k} = (\sqrt{5}\hat{\imath} + 2\hat{\jmath})(\pi/3) \times 10^7$ and because the phase is $\vec{k} \cdot \vec{r} - \omega t$ rather than $\vec{k} \cdot \vec{r} + \omega t$ the wave moves in the direction of $\vec{k}$. (d) $\vec{k} \cdot \vec{k} = (\pi \times 10^7)^2$, $k = \pi \times 10^7$ m^{-1} and $\lambda = 2\pi/k = 200$ nm. (e) $\omega = 9.42 \times 10^{15}$ rad/s and $\nu = \omega/2\pi = 1.5 \times 10^{15}$ Hz. (f) $v = \nu\lambda = 3.00 \times 10^8$ m/s.

3.6 (a) The field is linearly polarized in the y-direction and varies sinusoidally from zero at $z = 0$ to zero at $z = z_0$. (b) Using the wave equation

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0,$$

$$\left[-k^2 - \frac{\pi^2}{z_0^2} + \frac{\omega^2}{c^2}\right] E_0 \sin\frac{\pi z}{z_0}\cos(kx - \omega t) = 0$$

and since this is true for all x, z, and t each term must equal zero and so $k = (\omega/c)\sqrt{1 - (c\pi/\omega z_0)^2}$. (c) Moreover, $v = \omega/k = c/\sqrt{1 - (c\pi/\omega z_0)^2}$.

3.7 (a) $c = \nu\lambda$, so $\nu = c/\lambda = (3 \times 10^8 \text{ m/s})/(550 \times 10^{-9} \text{ m}) = 5.45 \times 10^{14}$ Hz. (b) $\omega = 2\pi\nu = 2\pi(5.45 \times 10^{14} \text{ Hz}) = 3.43 \times 10^{15}$ rad/s; $k = 2\pi/\lambda = 2\pi/(550 \times 10^{-9} \text{ m}) = 1.14 \times 10^{-7} \text{ m}^{-1}$. (c) $E_0 = cB_0$, so $B_0 = E_0/c = (600 \text{ V/m})/(3 \times 10^8 \text{ m/s}) = 2 \times 10^{-6} \text{ V-s/m}^2 = 2 \times 10^{-6}$ T. (d) $E(y,t) = E_0 \sin(ky - \omega t + \epsilon)$; $E(0,0) = 0 = E_0 \sin(\epsilon)$, $\epsilon = 0$; $B(y,t) = B_0 \sin(ky - \omega t + \epsilon)$; $B(0,0) = 0 = B_0 \sin(\epsilon)$, $\epsilon = 0$; $E(y,t) = (600 \text{ V/m}) \sin((1.14 \times 10^{-7} \text{ m}^{-1})y - (3.43 \times 10^{15} \text{ rad/s})t)$; $B(y,t) = (2 \times 10^6 T) \sin((1.14 \times 10^{-7} \text{ m}^{-1})y - (3.43 \times 10^{15} \text{ rad/s})t)$.

3.8 By Gauss' law, $E = \sigma/\epsilon_0$, where $\sigma = q/A$ is the surface charge density. Putting the average value of this electric field into $u_E = \epsilon_0 E^2/2$ gives $u_E = \sigma^2/8\epsilon_0$.

3.9 $u_B = B^2/2\mu_0$; $c = 1/\sqrt{\epsilon_0\mu_0}$, so $c^2\epsilon_0 = 1/\mu_0$. $u_B = c^2\epsilon_0 B^2/2$; $E = cB$, so $u_B = \epsilon_0(cB)^2/2 = \epsilon_0 E^2/2 = u_E$.

3.10 $\langle\cos^2(\vec{k}\cdot\vec{r} - \omega t)\rangle = (1/T)\int_t^{t+T} \cos^2(\vec{k}\cdot\vec{r} - \omega t')\,dt'$. Let $\vec{k}\cdot\vec{r} - \omega t' = x$; then $\langle\cos^2(\vec{k}\cdot\vec{r} - \omega t)\rangle = -(1/\omega T)\int \cos^2 x\,dx = -(1/2\omega T)\int(1 + \cos 2x)\,dx = -(1/2\omega T)[x + 0.5\sin 2x]_{\vec{k}\cdot\vec{r}-\omega t}^{\vec{k}\cdot\vec{r}-\omega(t+T)}$. Similarly use $\langle\sin^2(\vec{k}\cdot\vec{r} - \omega t)\rangle = (1/2)\langle 1 - \cos 2(\vec{k}\cdot\vec{r} - \omega t)\rangle$ and $\langle\sin(\vec{k}\cdot\vec{r} - \omega t)\cos(\vec{k}\cdot\vec{r} - \omega t)\rangle = (1/2)\langle\sin 2(\vec{k}\cdot\vec{r} - \omega t)\rangle$.