

in air

$$0.95/\lambda_0 = 0.19 \times 10^7;$$

total 2,050,000 waves.

$$OPD = [(1.5)(0.05) + (1)(0.95)] - (1)(1),$$

$$OPD = 1.025 - 1.000 = 0.025 \text{ m},$$

$$\Lambda/\lambda_0 = 0.025/500 \text{ nm} = 5 \times 10^4 \text{ waves.}$$

$$7.6 \quad OPL_B = nx = (1.00)(100 \text{ cm}) = 100 \text{ cm} = 1.00 \text{ m.}$$

$$OPL_A = \sum_i n_i x_i = (1.00)(89 \text{ cm}) + 2(1.52)(0.5 \text{ cm}) \\ + (1.33)(10 \text{ cm}) = 103.82 \text{ cm} = 1.0382 \text{ m.}$$

$$\Lambda = OPL_A - OPL_B = 1.0382 - 1.00 = .00382 \text{ m.}$$

$$(7.16) \quad \delta = k_0 \Lambda = (2\pi/\lambda_0)\Lambda = 2\pi(3.82 \times 10^{-3} \text{ m})/5.00 \times 10^{-9} \text{ m} \\ = 7.64 \times 10^6 \pi.$$

An integer multiple of  $2\pi$ , so waves are in phase.

$$7.7 \quad E_1 = E_{01} \sin[\omega t - k(x + \Delta x)], \text{ so } \alpha_1 = -k(x + \Delta x). \quad E_2 = E_{01} \sin[\omega t - kx], \\ \text{so } \alpha_2 = -kx.$$

$$(7.9) \quad E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos(\alpha_2 - \alpha_1) \\ = E_{01}^2 + E_{01}^2 + 2E_{01}^2 \cos(-kx - (-k(x + \Delta x))) = 2E_{01}^2 (1 + \cos k\Delta x) \\ = 2E_{01}^2 (\cos(0) + \cos(k\Delta x)) = 4E_{01}^2 \cos^2(k\Delta x/2),$$

(see Problem 7.2),

$$E_0 = 2E_{01} \cos(k\Delta x/2). \\ (7.10) \quad \tan \alpha = \frac{E_{01} \sin \alpha_1 + E_{02} \sin \alpha_2}{E_{01} \cos \alpha_1 + E_{02} \cos \alpha_2} \\ = \frac{E_{01} \sin(-k(x + \Delta x)) + E_{01} \sin(-kx)}{E_{01} \cos(-k(x + \Delta x)) + E_{01} \cos(-kx)}$$

$$\begin{aligned}
&= \frac{2 \sin \frac{1}{2}(-k(x + \Delta x) - kx) \cos \frac{1}{2}(-k(x + \Delta x) + kx)}{2 \cos \frac{1}{2}(-k(x + \Delta x) - kx) \cos \frac{1}{2}(-k(x + \Delta x) + kx)} \\
&= \tan(-kx - (k\Delta x/2)), \quad \alpha = -k(x + (\Delta x/2)).
\end{aligned}$$

7.8  $E = E_1 + E_2 = E_{01} \{ \sin[\omega t - k(x + \Delta x)] + \sin(\omega t - kx) \}$ . Since

$$\begin{aligned}
\sin \beta + \sin \gamma &= 2 \sin(1/2)(\beta + \gamma) \cos(1/2)(\beta - \gamma), \\
E &= 2E_{01} \cos(k\Delta x/2) \sin[\omega t - k(x + \Delta x/2)].
\end{aligned}$$

7.9  $E = E_0 \operatorname{Re}[e^{i(kx+\omega t)} - e^{i(kx-\omega t)}] = E_0 \operatorname{Re}[e^{ikx} 2i \sin \omega t]$   
 $= E_0 \operatorname{Re}[2i \cos kx \sin \omega t - 2 \sin kx \sin \omega t] = -2E_0 \sin kx \sin \omega t.$

Standing wave with node at  $x = 0$ .

7.10  $E_i = 3 \cos \omega t = 3\angle 0$ , ( $\alpha_1 = 0$ ).  $E_2 = 4 \sin \omega t$ , but  $\sin \theta = \cos(\theta - \pi/2)$ , so

$$E_2 = 4 \cos(\omega t - \pi/2) = 4\angle -\pi/2. \quad E_3 = E_1 + E_2.$$

$$\begin{aligned}
E_{3o}^2 &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos(\alpha_2 - \alpha_1) \\
&= 9 + 16 + 2(3)(4) \cos(-\pi/2), \quad E_{3o} = 5.
\end{aligned}$$

$$\begin{aligned}
(7.10) \quad \tan \alpha &= (E_{01} \sin \alpha_1 + E_{02} \sin \alpha_2) / (E_{01} \cos \alpha_1 + E_{02} \cos \alpha_2) \\
&= (3(0) + 4(-1)) / (3(1) + 4(0)) = -4/3; \quad \alpha = -53^\circ,
\end{aligned}$$

so  $\varphi = 53^\circ = 0.93$  rad. Note that  $\alpha_1 < \varphi$ , so  $E_1$  leads  $E_3$ .

7.11 By Faraday's law,  $\partial E / \partial x = -\partial B / \partial t$ . Integrate to get

$$\begin{aligned}
B(x, t) &= - \int (\partial E / \partial x) dt = -2E_0 k \cos kx \int \cos \omega t dt \\
&= -2E_0(k/\omega) \cos kx \sin \omega t.
\end{aligned}$$

But  $E_0 k / \omega = E_0 / c = B_0$ ; thus  $B(x, t) = -2B_0 \cos kx \sin \omega t$ .

7.12 Fringes are spaces  $\lambda/2$  vertically.

$$\sin \theta = (\text{fringes/cm}) \text{ vertical} / (\text{fringes/cm}) \text{ on film};$$

- 7.38** By analogy with Eq. (7.61),  $A(\omega) = (\Delta t/2)E_0 \text{sinc}(\omega_p - \omega)\Delta t/2$ . From Table 1,  $\text{sinc}(\pi/2) = 63.7\%$ . Not quite 50% actually,  $\text{sinc}(\pi/1.65) = 49.8\%$ .  $|(\omega_p - \omega)\Delta t/2| < \pi/2$  or  $-\pi/\Delta t < \omega_p - \omega < \pi/\Delta t$ ; thus appreciable values of  $A(\omega)$  lie in a range  $\Delta\omega \sim 2\pi/\Delta t$  and  $\Delta\nu\Delta t \sim 1$ . Irradiance is proportional to  $A^2(\omega)$ , and  $[\text{sinc}(\pi/2)]^2 = 40.6\%$ .
- 7.39**  $\Delta x_c = c\Delta t_c$ ,  $\Delta x_c \sim c/\Delta\nu$ . But  $\Delta\omega/\Delta k_0 = \bar{\omega}/\bar{k}_0 = c$ ; thus  $|\Delta\nu/\Delta\lambda_0| = \bar{\nu}/\bar{\lambda}_0$ ,  $\Delta x_c \sim c\bar{\lambda}_0/\Delta\lambda_0\bar{\nu}$ ,  $\Delta x_c \sim \bar{\lambda}_0^2/\Delta\lambda_0$ . Or try using the uncertainty principle:  $\Delta x \sim h/\Delta p$  where  $p = h/\lambda$  and  $\Delta\lambda_0 \ll \bar{\lambda}_0$ .
- 7.40**  $\Delta x_c = c\Delta t_c = 3 \times 10^8 \text{ m/s} \cdot 10^{-8} \text{ s} = 3 \text{ m}$ .  
 $\Delta\lambda_0 \sim \lambda_0^2/\Delta x_c = (500 \times 10^{-9} \text{ m})^2/3 \text{ m}$ ,  
 $\Delta\lambda_0 \sim 8.3 \times 10^{-14} \text{ m} = 8.3 \times 10^{-5} \text{ nm}$ ,  
 $\Delta\lambda_0/\bar{\lambda}_0 = \Delta\nu/\bar{\nu} = 8.3 \times 10^{-5}/500 = 1.6 \times 10^{-7} \sim 1 \text{ part in } 10^7$ .
- 7.41**  $\Delta\nu = 54 \times 10^3 \text{ Hz}$ ;  $\Delta\nu/\bar{\nu} = (54 \times 10^3)(10,600 \times 10^{-9} \text{ m})/(3 \times 10^8 \text{ m/s})$   
 $= 1.91 \times 10^{-9}$ .  $\Delta x_c = c\Delta t_c \sim c/\Delta\nu$ ,  $\Delta x_c \sim (3 \times 10^8)/(54 \times 10^3)$   
 $= 5.55 \times 10^3 \text{ m}$ .
- 7.42**  $\Delta\nu/\nu = 2/10^{10}$ ;  $c = \nu\lambda$ , so  

$$\nu = c/\lambda = 3 \times 10^8 \text{ m/s}/632.8 \times 10^{-9} \text{ m} = 4.74 \times 10^{14} \text{ Hz}$$
(7.64)  $\Delta\ell_c = c\Delta t_c$ .
- Frequency range is  $\pm 2(4.74 \times 10^4 \text{ Hz})$  or  $9.48 \times 10^4 \text{ Hz}$ , so  
 $\Delta t \simeq 1.05 \times 10^{-5} \text{ s}$ .  $\Delta\ell_c = (3 \times 10^8 \text{ m/s})(1.05 \times 10^{-5} \text{ s}) = 3.15 \times 10^3 \text{ m}$ .
- 7.43**  $\Delta x_c = c\Delta t_c = 3 \times 10^8 \times 10^{-10} = 3 \times 10^{-2} \text{ m}$ ,  $\Delta\nu \sim 1/\Delta t_c = 10^{10} \text{ Hz}$ ,  
 $\Delta\lambda_0 \sim \bar{\lambda}_0^2/\Delta x_c$  (see Problem 7.35),

**7.38** By analogy with Eq. (7.61),  $A(\omega) = (\Delta t/2)E_0 \text{sinc}(\omega_p - \omega)\Delta t/2$ . From Table 1,  $\text{sinc}(\pi/2) = 63.7\%$ . Not quite 50% actually,  $\text{sinc}(\pi/1.65) = 49.8\%$ .  $|(\omega_p - \omega)\Delta t/2| < \pi/2$  or  $-\pi/\Delta t < \omega_p - \omega < \pi/\Delta t$ ; thus appreciable values of  $A(\omega)$  lie in a range  $\Delta\omega \sim 2\pi/\Delta t$  and  $\Delta\nu\Delta t \sim 1$ . Irradiance is proportional to  $A^2(\omega)$ , and  $[\text{sinc}(\pi/2)]^2 = 40.6\%$ .

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