Chapter 10 Solutions

10.1 $(R+\ell)^2 = R^2 + a^2$; therefore $R = (a^2 - \ell^2)/2\ell \approx a^2/2\ell$, $\ell R = a^2/2$, so for $\lambda \ll \ell$, $\lambda R \gg a^2/2$. Therefore $R = (1 \times 10^{-3})^2 10/2\lambda = 10$ m.

10.2 $E_0/2 = R\sin(\delta/2)$, $E = 2R\sin(N\delta/2)$ chord length;

$$E = [E_0 \sin(N\delta/2)]/\sin(\delta/2), \quad I = E^2.$$

10.3 A "constant" phase shift is added due to the angle of the incident wave reaching the ends of the slit at different phase, so that (10.11) becomes $r = R - y(\sin\theta - \sin\theta_i) + \ldots$. This constant carries through the integration, so that the definition of β in 10.18 (or 10.14) becomes $\beta = (kb/2)(\sin\theta - \sin\theta_i)$.

10.4 $d\sin\theta_m = m\lambda$, $\theta = N\delta/2 = \pi$, $7\sin\theta = (1)(0.21)$, $\delta = 2\pi/N = kd\sin\theta$, $\sin\theta = 0.03$ so $\theta = 1.7°$. For $\sin\theta = 0.0009$, $\theta = 3$ min.

10.5 Converging spherical wave in image space is diffracted by the exit pupil.

10.6 $\beta = \pm\pi$, $\sin\theta = \pm\lambda/b \approx \theta$, $L\theta \approx \pm L\lambda/b$, $L\theta \approx \pm f_2\lambda/b$.

10.7 Far field if $R > b^2/\lambda$, $b^2/\lambda = (1 \times 10^{-4}\text{ m})^2/(4.619 \times 10^{-7}\text{ m})^2 = 0.02$. *Yes*, far field. $\sin\theta_1 = \lambda/b$.

$$\theta_1 = \sin^{-1}(\lambda/b) = \sin^{-1}(4.619 \times 10^{-7}\text{ m}/1 \times 10^{-4}\text{ m}) = 0.26°.$$

Angular width $= 2\theta_1 = 0.52°$.

10.8 $b\sin\theta_m = m\lambda$, so,

$$b = m\lambda/\sin\theta_m = 10(1.1522 \times 10^{-6}\text{ m})/\sin(6.2°) = 1.07 \times 10^{-4}\text{ m}.$$

In water, $b \sin \theta_m = m\lambda$, where $\lambda = n_{\text{air}}\lambda_o/n_{\text{water}}$.

$$\sin \theta_m = mn_{\text{air}}\lambda_o/n_{\text{water}}/b;$$

$$\theta_m = \sin^{-1}[10(1.00029)(1.1522 \times 10^{-6} \text{ m})/(1.33)(1.07 \times 10^{-4} \text{ m})]$$

$$= 4.7°.$$

10.9 $\lambda = (20 \text{ cm}) \sin 36.87° = 12 \text{ cm}$.

10.10 $\alpha = (ka/2) \sin \theta$, $\beta = (kb/2) \sin \theta$. $a = mb$, $\alpha = m\beta$, $\alpha = m2\pi$, N = number of fringes $= \alpha/\pi = m2\pi/\pi = 2m$.

10.11 Is $R > b^2/\lambda$?, b = slit width.

$$b^2\lambda = (1 \times 10^{-4} \text{ m})^2/(5 \times 10^{-7} \text{ m}) = .02 \text{ m} \ll 2.5 \text{ m}.$$

Fraunhofer.

(Half) angular width of central maximum from

$$\beta = \pi = (kb/2) \sin \theta_1.$$

$$\sin \theta_1 = 2\pi/kb = \lambda/b = (5 \times 10^{-7} \text{ m})/(1 \times 10^{-4} \text{ m}); \quad \theta_1 = 0.29°.$$

To what order Young's fringe does θ_1 correspond?

$$\alpha = m'\pi = (ka/2) \sin \theta_1. \quad m' = (ka/2\pi) \sin \theta_1 = (a/\lambda) \sin \theta_1$$

$$= (2 \times 10^{-4} \text{ m})/(5 \times 10^{-7} \text{ m}) \sin(0.29°) = 2.$$

So there are 4 "Young's Fringes" in the central maximum.

10.12 $\alpha = 3\pi/2N = \pi/2$, $I(\theta) = I(0)[(\sin \beta)/\beta]^2/N^2$ and $I/I(0) \approx 1/9$.

10.13 (10.17) $I(\theta) = I(0)(\sin \beta/\beta)^2$, where $\beta \equiv (kD/2) \sin \theta$. "Miniscule Area" corresponds to the limit $D \to 0$. As $D \to 0$, $\beta \to 0$, so

$$\lim_{D \to 0} I(\theta) = \lim_{\beta \to 0} (I(0)(\sin \beta/\beta)^2);$$

As $\beta \to 0$, $\text{sinc}(\beta) \to 1$, so $\lim_{D\to 0} I(\theta) = I(0)$, i.e., same in all directions.

10.14 (from 10.41) $\tilde{E} \propto \int\int e^{ik(Yy+Zz)/R}dS$. and $I(Y,Z) \propto \langle \tilde{E}^2 \rangle$. If $\tilde{E}$ is an even function of (Y,Z), $\tilde{E}(-Y,-Z) = \tilde{E}(Y,Z)$. If $\tilde{E}$ is an odd function of (Y,Z), $\tilde{E}(-Y,-Z) = -\tilde{E}(Y,Z)$, but $I(-Y,-Z) = I(Y,Z)$.

10.15 If the aperture is symmetrical about a line, the pattern will be symmetrical about a line parallel to it. Moreover, the pattern will be symmetrical about yet another line perpendicular to the aperture's symmetry axis. This follows from the fact that Fraunhofer patterns have a center of symmetry.

10.16 For the solution to this problem, please refer to the textbook.

10.17 Three parallel short slits.

10.18 Two parallel short slits.

10.19 An equilateral triangular hole.

10.20 A cross-shaped hole.

10.21 The E-field of a rectangular hole.

10.22 From section 10.2.5, first "ring" (maximum) occurs for $u = kaq/R = 5.14$. Interpolating from Table 10.1, $J_1(5.14) \simeq -0.33954$

From (10.55) $I/I(0) = \left[\frac{2J_1(u)}{u}\right]^2 = \left[\frac{2(-0.33954)}{5.14}\right]^2 = 0.0175$

10.23 From Eq. (10.58), $q_1 \approx 1.22(f/D)\lambda \approx \lambda$.

10.24 For the solution to this problem, please refer to the textbook.

10.25 (10.57) $q_1 = 1.22(R\lambda/2a)$

$$= 1.22[(3.76 \times 10^8 \text{ m})(6.328 \times 10^{-7} \text{ m})]/2[1 \times 10^{-3} \text{ m}]$$

$$= 1.45 \times 10^5 \text{ m}.$$

9.4 A bulb at S would produce fringes. We can imagine it as made up of a very large number of incoherent point sources. Each of these would generate an independent pattern, all of which would then overlap. Bulbs at S_1 and S_2 would be incoherent and could not generate detectable fringes.

9.5 $y_m = sm\lambda/a \approx 14.5 \times 10^{-2}$ m and $\lambda = 0.0145$ m: $\nu = v/\lambda = 23.7$ kHz. This is Young's Experiment with the sources out-of-phase.

9.6 This is comparable to the "two-slit" configuration, (Figure 9.8), so we can use (9.29) $a \sin\theta_m = m\lambda$ (θ_m may not be "small"). Let $m = 1$, $\sin\theta = y/(s^2+y^2)^{1/2}$, so,

$$ay = \lambda(s^2+y^2)^{1/2}; \quad (a^2-\lambda^2)y^2 = \lambda^2 s^2;$$

$$y = \lambda s/(a^2-y^2)^{1/2}. \quad c = v\lambda,$$

so $\lambda = c/v = (3\times10^8 \text{ m/s})/(1.0\times10^6 \text{ Hz}) = 300$ m.
$y = (300 \text{ m})(2000 \text{ m})/((600 \text{ m})^2 - (300 \text{ m})^2)^{1/2} = 1.15\times10^3$ m

9.7 (a) $r_1 - r_2 = \pm\lambda/2$, hence $a\sin\theta_1 = \pm\lambda/2$ and

$$\theta_1 \approx \pm\lambda/2a = \pm(1/2)(632.8\times10^{-9} \text{ m})/(0.220\times10^{-3} \text{ m})$$
$$= \pm1.58\times10^{-3} \text{ rad},$$

or since

$$y_1 = s\theta_1 = (1.00 \text{ m})(\pm1.58\times10^{-3} \text{ rad}) = \pm1.58 \text{ mm}.$$

(b) $y_5 = s5\lambda/a = (1.00 \text{ m})5(632.8\times10^{-9})/(0.200\times10^{-3} \text{ m}) = 1.582\times10^{-2}$ m. (c) Since the fringes vary as cosine-squared and the answer to (a) is half a fringe width, the answer to (b) is 10 times larger.

9.8 θ_m is "small," so we can use (9.28) $\theta_m = m\lambda/a$, θ_m is radian,

$$a = m\lambda/\theta_m = [4(6.943\times10^{-7} \text{ m})]/[1^\circ(2\pi \text{ rad}/360^\circ)] = 1.59\times10^{-4} \text{ m}.$$

9.9 $\Delta y \simeq (s/a)\lambda$, so,

$$s = a\Delta y/\lambda = [(1.0\times10^{-4} \text{ m})(10\times10^{-3} \text{ m})]/(4.8799\times10^{-7} \text{ m}) = 2.05 \text{ m}.$$

9.10 (9.28) $\theta_m = m\lambda/a$. Want $\theta_{1,\text{red}} = \theta_{2,\text{violet}}$; $(1)\lambda_{\text{red}}/a = (2)\lambda_{\text{violet}}/a$; $\lambda_{\text{violet}} = 390$ nm.

9.11 Follow section (9.3.1), except that (9.26) becomes $r_1 - r_2 = (2m' - 1)(\lambda/2)$ for destructive interference, where $m' = \pm1, \pm2, \ldots$, so that $(2m' - 1)$ is an odd integer. This leads to an expression equivalent to (9.28), $\theta_{m'} = (2m - 1)\lambda/2a$.

9.12 Follow section (9.3.1), except that (9.26) becomes $r_1 - r_2 + \Lambda = m\lambda$, where $\Lambda =$ Optical path differences in beam. Following r_1, $\Lambda = nd$ (for θ_m "small").

$$(r_1 - r_2) = m\lambda - \Lambda; \quad a\theta_m = m\lambda - nd; \quad \theta_m = (m\lambda - nd)/a.$$

9.13 As in section (9.3.1), we have constructive interference when $OPD = m\lambda$. There is an added OPD due to the angle, θ, of the plane wave equal to $a\sin\theta$, so (9.26) becomes $r_1 - r_2 + a\sin\theta = m\lambda$. (9.24) $\theta_m \simeq y/s$ and (9.25) $r_1 - r_2 \simeq ay/s$ are unchanged, for small θ_m so $r_1 - r_2 = m\lambda - a\sin\theta = a(y/s) = a\theta_m$; $\theta_m = (m\lambda/a) - \sin\theta$.

9.14 (9.27) $y_m = (s/a)m\lambda$; $y_{1,\text{red}} = [(2.0 \text{ m})/(2.0 \times 10^{-4} \text{ m})](1)(4 \times 10^{-7} \text{ m})$
$= 4.0 \times 10^{-3}$ m.
$y_{1,\text{violet}} = [(2.0 \text{ m})/(2.0 \times 10^{-4} \text{ m})](2)(6 \times 10^{-7} \text{ m}) = 12.0 \times 10^{-3}$ m.
Distance $= 8.0 \times 10^{-3}$ m.

9.15 $r_2^2 = a^2 + r_1^2 - 2ar_1\cos(90° - \theta)$. The contribution to $\cos\delta/2$ from the third term in the Maclaurin expansion will be negligible if

$$(k/2)(a^2\cos^2\theta/2r_1) \ll \pi/2; \qquad \text{therefore} \qquad r_1 \ll a^2/\lambda.$$

9.16 $E = mv^2/2$; $v = 0.42 \times 10^6$ m/s; $\lambda = h/mv = 1.73 \times 10^{-9}$ m; $\Delta y = s\lambda/a = 3.46$ mm.

9.21 From Problem 9.19, $a = 2d(n-1)\alpha$; $s = 2d$, so $d = 1m$.

$$\Delta y = (s/a)\lambda = s\lambda/2d(n-1)\alpha; \quad \alpha = s\lambda/2d(n-1)\Delta y$$
$$= [(2m)(5.00 \times 10^{-7} \text{ m})]/[2(1 \text{ m})(1.5-1)(5 \times 10^{-4} \text{ m})] = 0.002 \text{ rad.}$$

9.22 $\Delta y = s\lambda_0/2d\alpha(n-n')$.

9.23 $\Delta y = (s/a)\lambda$, $a = 10^{-2}$ cm, $a/2 = 5 \times 10^{-3}$ cm.

9.24 $\delta = k(r_1 - r_2) + \pi$ Lloyd's mirror,

$$\delta = k(a/2\sin\alpha - [\sin(90^\circ - 2\alpha)]a/2\sin\alpha) + \pi,$$
$$\delta = ka(1-\cos 2\alpha)/2\sin\alpha + \pi,$$

maximum occurs for $\delta = 2\pi$ when $\sin\alpha(\lambda/a) = (1-\cos 2\alpha) = 2\sin^2\alpha$. First maximum $\alpha = \sin^{-1}(\lambda/2a)$.

9.25 E_{1r} is reflected once. $E_{1r} = E_{oi}\, r_{\theta=0}$ (see 4.47) $= E_{oi}(n-1)/(n+1) = E_{oi}(1.52-1)/(1.52+1) = 0.206E_{oi}$.
E_{2r} is transmitted once, reflected once, then transmitted.
$E_{2r} = E_{oi}(t_{\theta=0})(r'_{\text{glass-air}})(t'_{\text{glass-air}}) = E_{oi}[2/(1+n)][(1-n)/(1+n)][2n/(n+1)] = 4n(1-n)/(n+1)^3 = E_{oi}[4(1.52)(1-1.52)]/(1+1.52)^3 = -0.198E_{oi}$, (see 4.48) (– indicates π phase changed).
E_{3r} is transmitted, reflected 3 times (internally), and then transmitted.

$$E_{3r} = E_{oi}t(r')^3t' = E_{oi}[2/(1+n)][(1-n)/(1+n)]^3[(2n)/(n+1)]$$
$$= [4n(1-n)^3]/(n+1)^5 = E_{oi}[4(1.52)(1-1.52)^3]/(1.52+1)^5$$
$$= -0.008E_{oi}$$

for water in air.

$$E_{1r} = E_{oi}(1.333-1)/(1.333+1) = 0.143E_{oi}.$$
$$E_{2r} = E_{oi}[4(1.333)(1-1.333)]/(1+1.333)^3 = -0.140E_{oi}.$$
$$E_{3r} = E_{oi}[4(1.333)(1-1.333)^3]/(1.333+1)^5 = -0.003E_{oi}.$$

9.26 Here $1.00 < 1.34 < 2.00$, hence from Eq. (9.36) with $m = 0$, $d = (0 + 1/2)(633 \text{ nm})/2(1.34) = 118 \text{ nm}$.

9.27 (9.36) $d \cos\theta_t = (2m + 1)(\lambda_f)/4$ for a maximum at (near) normal incidence, and taking $m = D$ (lowest value)

$$d = \lambda f/4 = \lambda_o/4n = (5.00 \times 10^{-7} \text{ m})/4(1.36) = 9.2 \times 10^{-6} \text{ m}.$$

9.28 (9.37) $d \cos\theta_t = 2m(\lambda_f/4)$ for minimum reflection $= 2m(\lambda_o/n)$ at $\theta \simeq 0$, $\lambda_o = nd/2m = [(1.34)(550.0 \text{ nm})]/2 \text{ m} = 368.5(1/m) \text{ nm}$, $m = 1, 2, 3, \ldots$ or $\lambda_o = 368.5$ nm, 184.25 nm, 122.83 nm,. . . .

9.29 Eq. (9.37) $m = 2n_f d/\lambda_0 = 10{,}000$. A minimum, therefore central dark region.

9.30 The fringes are generally a series of fine jagged bands, which are fixed with respect to the glass.

9.31 $x^2 = d_1[(R_1 - d_1) + R_1] = 2R_1 d_1 - d_1^2$. Similarly $x^2 = 2R_2 d_2 - d_2^2$. $d = d_1 - d_2 = (x^2/2)(1/R_1 - 1/R_2)$, $d = m\lambda_f/2$. As $R_2 \to \infty$, x_m approaches Eq. (9.43).

9.32 (9.42) $x_m = [(m + 1/2)\lambda_f R]^{1/2}$, air film, $n_f = 1$, so $\lambda_f = \lambda_o$. $R = x_m^2/(m + 1/2)\lambda_o = (0.01 \text{ m})^2/(20.5)(5 \times 10^{-7} \text{ m}) = 9.76 \text{ m}$.

9.33 $\Delta x = \lambda_f/2\alpha$, $\alpha = \lambda_0/2n_f\Delta x$, $\alpha = 5 \times 10^{-5}$ rad$=$ 10.2 seconds.

9.34 (9.40) $\Delta x = \lambda_f/2\alpha$ for fringe separation where $\alpha = d/x$. $\Delta x = \lambda_f/2(d/x) = x\lambda_f/2d$. Number of fringes = (length)/(separation) $= x/\Delta x$ so,

$$x/\Delta x = 2d/\lambda_f = [2(7.618 \times 10^{-5} \text{ m})]/(5.00 \times 10^{-7} \text{ m}).$$

9.35 A motion of $\lambda/2$ causes a single fringe pair to shift past, hence $92\lambda/2 = 2.53 \times 10^{-5}$ m and $\lambda = 550$ nm.

9.36 $\Delta d = N(\lambda_o/2) = (1000)(5.00 \times 10^{-7} \text{ m})/2 = 2.50 \times 10^{-4} \text{ m}$.