4.53 Light incident from air to glass. θ_t increases as θ_i increases, so Maximum θ_t should correspond to Maximum θ_i.
(4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so, $\sin\theta_t = (n_i/n_t)\sin\theta_i$.
Maximum $\theta_i \leq 90°$ as $\theta_i \rightarrow 90°$, $\sin\theta_i \rightarrow 1$ so, $\sin\theta_t = n_i/n_t = \sin\theta_c$.

4.54 $1.00/2.417 = \sin\theta_c$; $\theta_c = 24°$ diamond refracts light back out and so looks brilliant.

4.55 $\sin 48.0° = (1.00/n)$; $n = 1.35$.

4.56
$$\theta_i = 45° \rightarrow \theta_c$$
$$\sin\theta_i = \frac{n_t}{n_i}, \quad \text{where} \quad n_t = 1$$
$$n = \frac{1}{\sin\ 45°} = 1.41$$

4.57 Light entering at glancing incidence is transmitted at the critical angle and those rays limit the cone of light reaching the fish; $\sin\theta_c = 1/1.333$; $\theta_c = 49°$ and the cone-angle is twice this or $98°$.

4.58 $\sin\theta_c = n_t/n_i$; $\theta_c = 59.1°$.

4.59 From Eq. (4.73) we see that the exponential will be in the form $k(x - vt)$, provided that we factor out $k_t \sin\theta_i/n_{ti}$, leaving the second term as $\omega n_{ti} t/k_t \sin\theta_i$, which must be $v_t t$. Hence $\omega n_t/(2\pi/\lambda_t) n_i \sin\theta_i = v_t$, and so $v_t = c/n_i \sin\theta_i = v_i/\sin\theta_i$.

4.60 From the defining equation, $\beta = k_t[(\sin^2\theta_i/n_{ti}^2) - 1]^{1/2} = 3.702 \times 10^6\ \text{m}^{-1}$, and since $y\beta = 1$, $y = 2.7 \times 10^{-7}$ cm.

4.61 The beam scatters off the wet paper and is mostly transmitted until the critical angle is attained, at which point the light is reflected back toward the source. $\tan\theta_c = (R/2)/d$, and so $n_{ti} = 1/n_i = \sin[\tan^{-1}(R/2d)]$.

4.62 $1.00029 \sin 88.7° = n \sin 90°$, $n = 1.00003$.

4.63 Let $\theta_i = \theta_p = \pi/2 - \theta_t$. Reflected beam is polarized if $r_\perp$ or $r_\parallel$ equal zero.
(4.43)

$$r_\parallel = \tan(\theta_i - \theta_t)/\tan(\theta_i + \theta_t) = \tan(\pi/2 - \theta_t - \theta_t)/\tan(\pi/2 - \theta_t + \theta_t)$$
$$= \tan(\pi/2 - 2\theta_t)/\tan(\pi/2).$$

But $\tan(\pi/2)$ is infinite, so $r_\parallel = 0$.

4.64 $\theta_i + \theta_t = 90°$ when $\theta_i = \theta_p$, $n_i \sin\theta_p = n_t \sin\theta_t = n_t \cos\theta_p$, $\tan\theta_p = n_t/n_i = 1.52$, $\theta_p = 56°40'$.

4.65 At θ_p, $r_\parallel = 0$. So from (4.38) $\frac{n_t}{\mu_t}\cos\theta_i - \frac{n_i}{\mu_i}\cos\theta_t = 0$. Recall (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$. (3.59) $n = \sqrt{\epsilon\mu/\epsilon_0\mu_0}$ and $\cos^2\theta = 1 - \sin^2\theta$. Approach: solve for $\tan\theta_p = \sin\theta_p/\cos\theta_p$ where $\theta_i = \theta_p$.

4.66 $\tan\theta_p = n_t/n_i = n_2/n_1$, $\tan\theta'_p = n_1/n_2$, $\tan\theta_p = 1/\tan\theta'_p$. $\sin\theta_p/\cos\theta_p = \cos\theta'_p/\sin\theta'_p$. Therefore $\sin\theta_p \sin\theta'_p - \cos\theta_p\cos\theta'_p = 0$, $\cos(\theta_p + \theta'_p) = 0$, so $\theta_p + \theta'_p = 90°$.

4.67 From Eq. (4.94), $\tan\gamma_r = r_\perp[E_{oi}]_\perp/r_\parallel[E_{oi}]_\parallel = (r_\perp/r_\parallel)\tan\gamma_i$ and from Eqs. (4.42) and (4.43)

$$\tan\gamma_r = -\frac{\cos(\theta_i - \theta_t)}{\cos(\theta_i + \theta_r)}\tan\gamma_i.$$

4.68 (4.56) $R = \left(\frac{E_{or}}{E_{oi}}\right)^2$ $E_{or}^2 = E_{or\parallel}^2 + E_{or\perp}^2$. $E_{oi}^2 = E_{oi\parallel}^2 + E_{oi\perp}^2$.
(4.34) $r_\perp \equiv \left(\frac{E_{or}}{E_{oi}}\right)_\perp$. (4.38) $r_\parallel \equiv \left(\frac{E_{or}}{E_{oi}}\right)_\parallel$.

$$R = \frac{E_{or\perp}^2 + E_{or\parallel}^2}{E_{oi\perp}^2 + E_{oi\parallel}^2}$$
$$= \frac{(E_{or\perp}/E_{oi\perp})^2}{1 + (E_{oi\parallel}/E_{oi\perp})^2}$$
$$+ \frac{(E_{or\parallel}/E_{oi\parallel})^2}{(E_{oi\perp}/E_{oi\parallel})^2} + 1$$

$$= \frac{r_\perp^2}{1+\cot^2\gamma_i} + \frac{r_\parallel^2}{\tan^2\gamma_i + 1}$$
$$= R_\perp \sin^2\gamma_i + R_\parallel \cos^2\gamma_i$$

$$(4.57) T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{ot}}{E_{oi}}\right)^2$$

as above, $\left(\frac{E_{ot}}{E_{oi}}\right)^2 = t_\perp^2 \sin^2\gamma_i + t_\parallel \cos^2\gamma_i$, and using (4.63, 4.64),

$$T_{\perp,\parallel} = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_{\perp,\parallel}^2.$$

$T = T_\perp \sin^2\gamma_i + T_\parallel \cos^2\gamma_i$.

4.69 Note that $\theta_e = 41.8°$. Note that $R_\perp$ increases steadily, while $R_\parallel$ has a minimum at $\theta_i \neq 0$.

4.70 $T_\perp = n_t t_\perp^2 \cos\theta_t / n_i \cos\theta_i$. From Eq. (4.44) and Snell's law,

$$T_\perp = \left(\frac{\sin\theta_i \cos\theta_t}{\sin\theta_t \cos\theta_i}\right)\left(\frac{4\sin^2\theta_t \cos^2\theta_i}{\sin^2(\theta_i+\theta_t)}\right) = \frac{\sin 2\theta_i \sin 2\theta_t}{\sin^2(\theta_i+\theta_t)}.$$

4.71 Use (4.62) and (4.43). $R_\parallel = r_\parallel^2 = \tan^2(\theta_i - \theta_t)/\tan^2(\theta_i+\theta_t) = [\sin^2(\theta_i-\theta_t)/\cos^2(\theta_i-\theta_t)] \times [\cos^2(\theta_i+\theta_t)/\sin^2(\theta_i+\theta_t)]$. Note that $R_\parallel$ and $T_\parallel$ have now the same denominator.

Use (4.61) and (4.42). $R_\perp = r_\perp^2 = \sin^2(\theta_i - \theta)/\sin^2(\theta_i+\theta_t)$. Note that $R_\perp$ and $T_\perp$ have the same denominator.

4.72 If Φ_i is the incident radiant flux or power and T is the transmittance across the first air-glass boundary, the transmitted flux is then $T\Phi_i$. From Eq. (4.68) at normal incidence the transmittance from glass to air is also

T. Thus a flux $T\Phi_i T$ emerges from the first slide, and $\Phi_i T^{2N}$ from the last one. Since $T = 1 - R$, $T_t = (1 - R)^{2N}$ from Eq. (4.67).

$$R = (0.5/2.5)^2 = 4\%, \quad T = 96\%, \quad T_t = (0.96)^6 \approx 78.3\%.$$

4.73 $T = I(y)/I_0 = e^{-\alpha y}$, $T_1 = e^{-\alpha}$, $T = (T_1)^y$. $T_t = (1 - R)^{2N}(T_1)^d$.

4.74 At $\theta_i = 0$, $R = R_\parallel = R_\perp = [(n_t - n_i)/(n_t + n_i)]^2$. As $n_{ti} \to 1$, $n_t \to n_i$ and clearly $R \to 0$. At $\theta_i = 0$, $T = T_\parallel = T_\perp 4n_t n_i/(n_t + n_i)^2$ and since $n_t \to n_i$, $\lim_{n_{ti}\to 1} T = 4n_i^2/(2n_i)^2 = 1$. From Problem 4.61 and the fact that as $n_t \to n_i$ Snell's law says that $\theta_t \to \theta_i$, we have

$$\lim_{n_{ti}\to 1} T_\parallel = \sin^2 2\theta_i / \sin^2 2\theta_i = 1, \qquad \lim_{n_{ti}\to 1} T_\perp = 1.$$

From Eq. (4.43) and the fact that $R_\parallel = r_\parallel^2$ and $\theta_t \to \theta_i$, $\lim_{n_{ti}\to 1} R_\parallel = 0$.

4.75 (4.34)

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{\cos\theta_i - n_{ti}\cos\theta_t}{\cos\theta_i + n_{ti}\cos\theta_t} = \frac{\cos\theta_i - n_{ti}\sqrt{1 - \sin^2\theta_t}}{\cos\theta_i + n_{ti}\sqrt{1 - \sin^2\theta_t}} = \frac{\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n_{ti}^2 + \sin^2\theta_i}}$$

$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} = \frac{n_{ti}\cos\theta_i - \sqrt{1 - \sin^2\theta_t}}{n_{ti}\cos\theta_i + \sqrt{1 - \sin^2\theta_t}} = \frac{n_{ti}^2\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_t}}{n_{ti}^2\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_t}}$$