Physics 451 **Supplementary Assignment** **Fall 2005**

Regarding energy conservation in numerical solutions of equations of motion.

For an object moving under the influence of conservative forces, we expect the total mechanical energy, E = K + U, to be constant with time. In our numerical solutions, we have truncation and round-off errors which lead to random (we hope) fluctuations in K and U and thus in the total energy E. If we imagine E(t) to be some sort of signal, then the random fluctuations play the role of noise in the signal.

The upshot is that we do not expect E to be identical from one time step to the next. However, we do expect the total energy averaged over long-ish subintervals in time to be constant.

For example, in the case of the Earth in orbit about the Sun, we might average the total energy over one quarter orbit as the subinterval. We might also compute an average deviation or a standard deviation of the total energy over that quarter orbit. We expect that over a complete orbit, or over many complete orbits, the sub-averages (as you might call them) are equal, within the limits of the standard deviations.

Here's a Problem: Take your total energy output from Problem 4.5 and verify that the total energy, E, is conserved by computing sub-averages and standard deviations. Do this for some different time steps, h, and discuss the resulting standard deviations of the total energy.