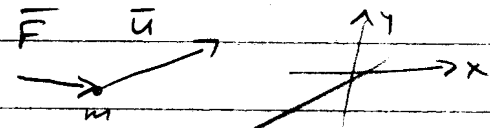
Phys 203

1. In the Lab frame:



$$\bar{F} = \frac{d\bar{p}}{dt}$$

As viewed in a frame moving with constant velocity $\bar{v}$ w.r.t. the Lab frame

$$\bar{p}' = \bar{p} + m\bar{v}$$

Take the derivation $\frac{d\bar{p}'}{dt} = \frac{d\bar{p}}{dt} + \overset{0}{\cancel{\frac{d}{dt} m\bar{v}}}$

Thus, $\frac{d\bar{p}'}{dt} = \frac{d\bar{p}}{dt}$ and $\bar{F} = \frac{d\bar{p}}{dt}$; $\bar{F} = \frac{d\bar{p}'}{dt}$

Newton's 2nd law is valid in both frames.

~~II.~~ If the "primed frame" is accelerating
1. w.r.t. the Lab-frame, then

$$\frac{d\bar{p}'}{dt} = \frac{d\bar{p}}{dt} + m\frac{d\bar{v}}{dt}$$

$$\frac{d\bar{p}'}{dt} = \bar{F} + m\frac{d\bar{v}}{dt}$$

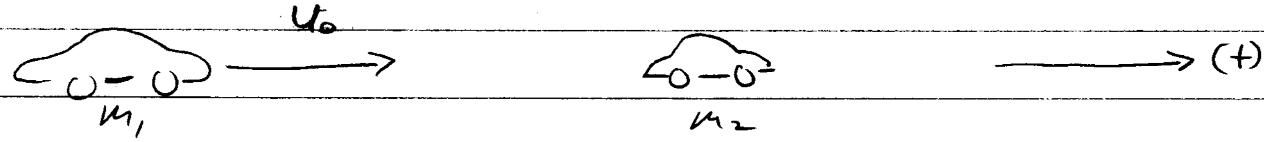
Newton's 2nd law would need

$$\bar{F} = \frac{d\bar{p}'}{dt} - m\frac{d\bar{v}}{dt}$$

So the 2nd Law is not valid. The net applied force does not equal the observed time rate of change of the particle's momentum.

~~1.~~ In the S-frame:

2



Initial momentum $p_i = m_1 u_0$

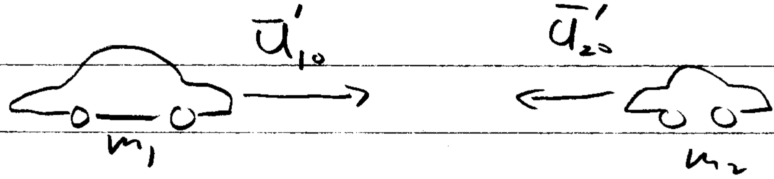
Final momentum $p_f = (m_1 + m_2) u_f$

The momentum is conserved in this frame, so we can solve for u_f by

$$p_f = p_i$$

$$u_f (m_1 + m_2) = m_1 u_0 \Longrightarrow u_f = u_0 \frac{m_1}{m_1 + m_2}$$

Now we must demonstrate that the momentum is conserved in the S' frame. The S' frame is moving at constant velocity $\bar{v}$ w.r.t. the S-frame.



The Initial momentum is

$$p_i' = m_1 u_{10}' + m_2 u_{20}'$$

The final momentum is

$$p_f' = (m_1 + m_2) u_f'$$

Well, first write u_{10}' + u_{20}' in terms of u_0 and v.

$$u_{10}' = u_0 - v \quad \text{and} \quad u_{20}' = -v$$

Thus $p_i' = m_1 (u_0 - v) + m_2 (-v) = m_1 u_0 - v (m_1 + m_2)$

2 Second, we transform the final velocity in the S-frame to the S'-frame

$$u_f' = u_f - v.$$

Compute $p_f' = (m_1+m_2)\, v_f' =$

$$= (m_1+m_2)(u_f - v)$$

Put in u_f in terms of u_0

$$p_f' = (m_1+m_2)\left(\frac{u_0 m_1}{m_1+m_2} - v\right)$$

$$= u_0 m_1 - (m_1+m_2)v$$

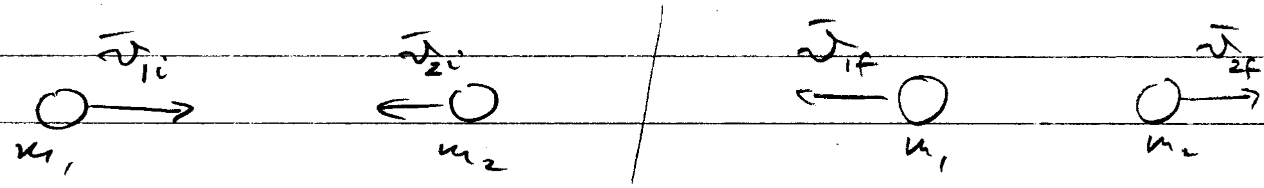
But this is the same as p_i'. That is $p_i' = p_f'$.

Notice, we don't even need to plug in the given numerical values.

Phys 203

3 #. For ~~elastic~~ collision, both the momentum and kinetic energy are conserved.

In the S-frame $\longrightarrow (+)$



Because the kinetic energy is conserved, we have $v_{2f} - v_{1f} = v_{1i} - v_{2i}$. (1)

Momentum is conserved

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} \quad (2)$$

Solve eqn (1) for v_{2f}

$$v_{2f} = v_{1f} + v_{1i} - v_{2i}$$, plug into (2)

$$m_1 v_{1f} + m_2 (v_{1f} + v_{1i} - v_{2i}) = m_1 v_{1i} + m_2 v_{2i}$$

Solve for v_{1f}

$$v_{1f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_2 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Finally plug this back into equation (1) to obtain v_{2f}.

3^64 In the S'- frame

$$v_{1i}' = v_{1i} - v \quad \text{and} \quad v_{2i}' = v_{2i} - v$$

What would be v_{1f}'? if $p_i' = p_f'$?

$$v_{1f}' = \frac{m_1 v_{1i}' + m_2 v_{2i}' - m_2 v_{1i}' + m_2 v_{2i}'}{m_1 + m_2}$$

Substitute for v_{1i}' and v_{2i}'

$$v_{1f}' = \frac{m_1 (v_{1i} - v) + m_2 (v_{2i} - v) - m_2 (v_{1i} - v) + m_2 (v_{2i} - v)}{m_1 + m_2}$$

$$v_{1f}' = v_{1f} - v$$

And
$$v_{2f}' = v_{1f}' + v_{1i}' - v_{2i}'$$
$$v_{2f}' = (v_{1f} - v) + (v_{1i} - v) - (v_{2i} - v)$$
$$v_{2f}' = v_{1f} + v_{1i} - v_{2i} - v$$
$$v_{2f}' = v_{2f} - v$$

The final velocities in the 2 frames are related by the same relative velocity, v.

i.e. $v_{1f}' = v_{1f} - v$ and
$v_{2f}' = v_{2f} - v$