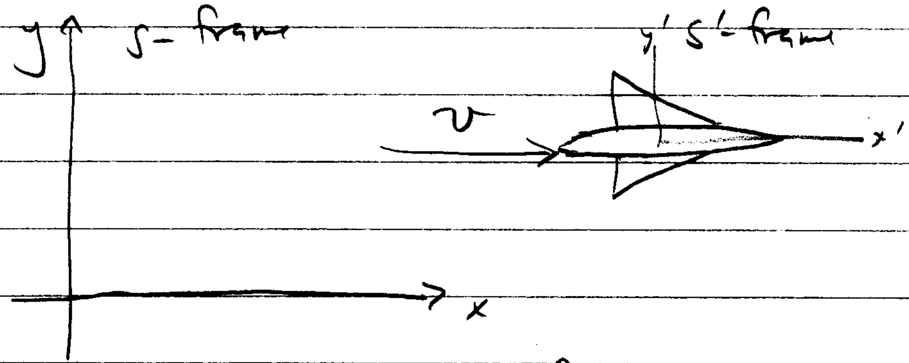
1-11 3rd ed

Phys 203



The proper time interval measured on the plane is $\Delta t' = 3600$ sec. The plane's speed in the S-frame is $v = 400$ m/sec.

An observer on the ground, in the S-frame, will measure a longer time interval, Δt.

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In this case, $v \ll c$, so approximate

$$\Delta t = \left(1 + \frac{v^2}{2c^2}\right)\Delta t'.$$ Plug in v + $\Delta t'$

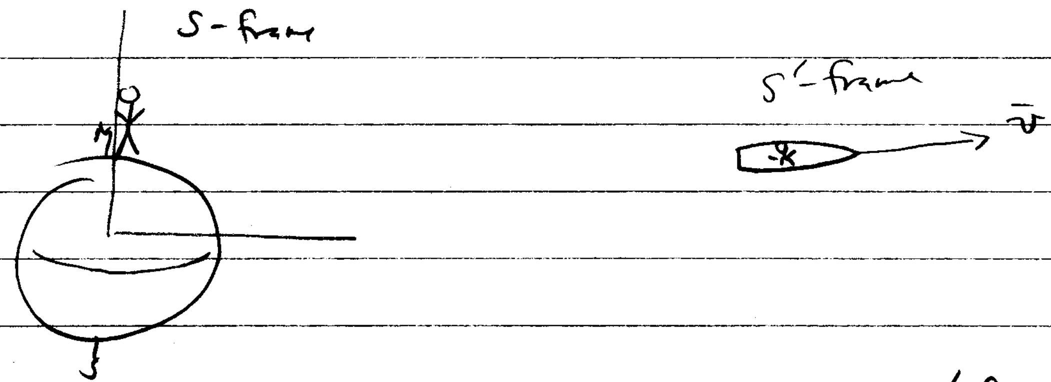
$$\Delta t = \left(1 + \frac{(400\text{ m/sec})^2}{2c^2}\right) 3600 \text{ sec}$$

$$\Delta t = \left(1 + 9 \times 10^{-13}\right) 3600 \text{ sec}$$

$$\Delta t - \Delta t' = 9 \times 10^{-13} (3600 \text{ sec}) \simeq 3 \times 10^{-9} \text{ sec.}$$

1.12 3rd ed

~~1.15~~



The astronaut in the rocket, in the S'-frame, has a heartbeat rate of 70 min^{-1}. The period of the beats is then $\Delta t' = 0.0143$ min. The rocket travels with speed $v = 0.90c$ in the S-frame.

a) In the rocket the astronaut's heart rate is 70 beats/min, 'cause it's being measured in the astronaut's rest frame.

b) Measured relative to the S-frame, the period of the heart beat in the rocket is

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{plug in } v \text{ \& } \Delta t'$$

$$\Delta t = \frac{0.0143 \text{ min}}{\sqrt{1 - .90^2}} = 0.328 \text{ min}$$

The number of beats per minute, then, is

$$\frac{1}{.328} = 30.5 \frac{\text{beats}}{\text{min.}}$$

3^{rd} ed.

1-13 Muon decay

$$N = N_0 e^{-t/\tau} \qquad \tau = 2.20 \mu s$$

$$N_0 = 5.0 \times 10^4$$

$$v = 0.95c$$

The proper time is $\Delta t' = \tau' = 2.20 \mu s$.

As observed in the Lab,

$$\Delta t = \tau = \gamma \tau' = \gamma \Delta t'$$

$$\Delta t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Delta t'$$

$$\Delta t = \frac{1}{\sqrt{1 - .95^2}} 2.20 \mu s$$

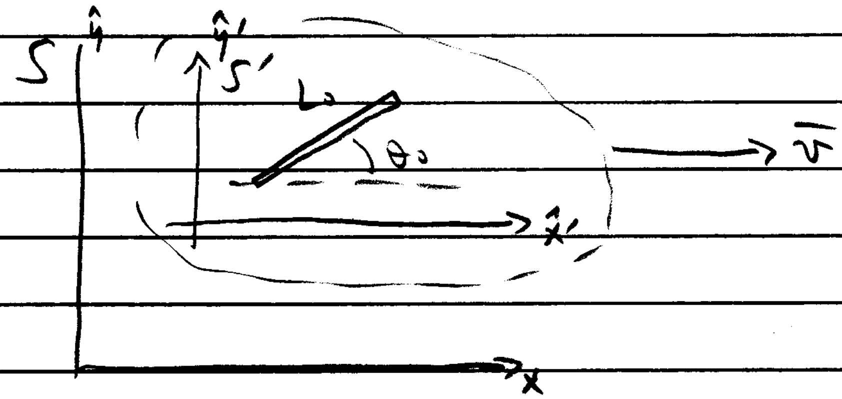
$$\Delta t = \frac{1}{.312} 2.20 \mu s = 7.04 \mu s$$

At $v = .95c$, the muons travel a distance of $3.0 \times 10^3 m$ in $t = 1.05 \times 10^{-5}$ sec.

Therefore $N = 5 \times 10^4 e^{-\frac{1.05 \times 10^{-5}}{7.04 \times 10^{-6}}}$

$$N = 1.13 \times 10^4$$

1-14 (third ed)



$\bar{v}$ is along $\hat{x}(\hat{x}')$.

In the S'-frame $L_0^2 = \Delta x'^2 + \Delta y'^2$ and
$\Delta x' = L_0 \cos\theta_0 \qquad \Delta y' = L_0 \sin\theta_0$

In the S-frame $\Delta x = \sqrt{1-\frac{v^2}{c^2}}\,\Delta x' = \sqrt{1-\frac{v^2}{c^2}}\,L_0\cos\theta_0$

but, $\Delta y = \Delta y'$.

$$L^2 = \Delta x^2 + \Delta y^2 = \left(1-\frac{v^2}{c^2}\right)(L_0\cos\theta_0)^2 + (L_0\sin\theta_0)^2$$

$$L^2 = L_0^2\left(\cos^2\theta_0 - \frac{v^2}{c^2}\cos^2\theta_0 + \sin^2\theta_0\right)$$

$$L^2 = L_0^2\left(1 - \frac{v^2}{c^2}\cos^2\theta\right)$$

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~~1.17~~

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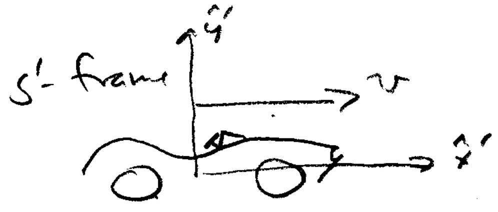
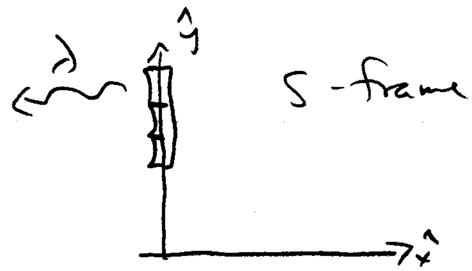




λ is the wavelength measured in the S-frame.
λ' is the wavelength measured in the S'-frame.

We've given that $\lambda' = 550\,nm$ and $\lambda = 650\,nm$.

Use $\frac{1}{f'} = \frac{1}{f}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$ or eqn 1.13

$$f' = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}}\, f \quad ; \quad f = \frac{c}{\lambda}$$

$$\frac{c}{\lambda'} = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}}\,\frac{c}{\lambda} \quad ; \text{ solve for } \frac{v}{c}$$

$$\left(\frac{\lambda}{\lambda'}\right)^2 = \frac{1+\frac{v}{c}}{1-\frac{v}{c}}$$

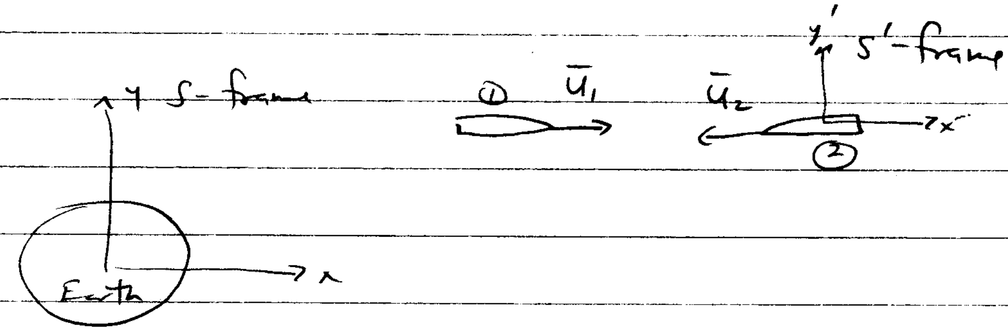
$$\left(\frac{\lambda}{\lambda'}\right)^2\left(1-\frac{v}{c}\right) = 1+\frac{v}{c}$$

$$\frac{v}{c}\left(-\left(\frac{\lambda}{\lambda'}\right)^2 - 1\right) = 1 - \left(\frac{\lambda}{\lambda'}\right)^2 \quad ; \quad \frac{\lambda}{\lambda'} = \frac{650}{550} = 1.18$$

$$\frac{v}{c} = \frac{1 - 1.18^2}{-1.18^2 - 1} = \frac{-.39}{-2.39}$$

$v = 0.163\,c \approx 10^8$ mph. The judge won't buy it.

1-19 3rd ed.



As observed in the S-frame on the Earth, the two rocket ships have velocities $\bar{u}_1$ and $\bar{u}_2$, where $\bar{u}_2 = -\bar{u}_1$. As viewed in the S'-frame, the first has a velocity of $\bar{u}_1' = +0.7c\,\hat{x}'$. The relative velocity of the two reference frames is $\bar{v} = \bar{u}_2$. [S-frame on Earth; S' frame on rocket②]

We want to solve for u_1 or u_2.

The velocity transformation of u_1 is

$$u_1' = \frac{u_1 - u_2}{1 - \frac{u_1 u_2}{c^2}}, \quad \text{but } u_2 = -u_1$$

$$u_1' = \frac{2u_1}{1 + \frac{u_1^2}{c^2}}, \quad \text{rearrange}$$

$$\frac{u_1^2}{c^2} - \frac{2u_1}{u_1'} + 1 = 0, \quad \text{use quadratic formula}$$

$$u_1 = \frac{\frac{2}{u_1'} \pm \sqrt{\frac{4}{u_1'^2} - \frac{4}{c^2}}}{\frac{2}{c^2}}$$

[the root > c is not physical]

$u_1 = 0.41c$, and $u_2 = -0.41c$

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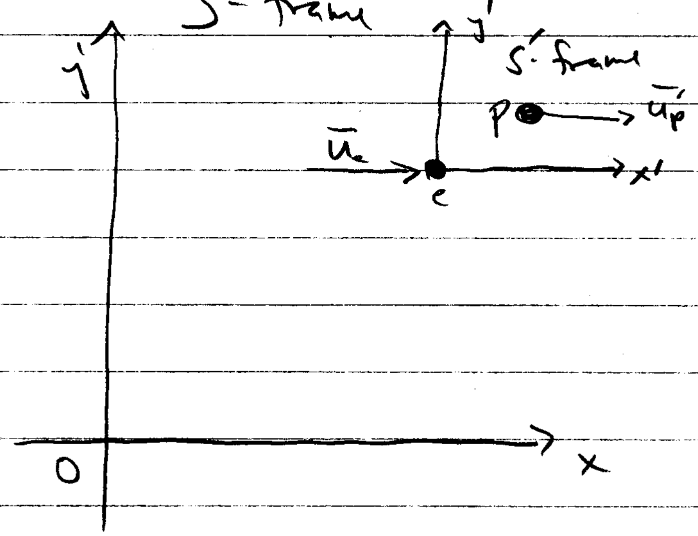

$u_e = 0.90c$

$u'_p = 0.70c$

The relative velocity of the 2 reference frames is $\bar{v} = \bar{u}_e$ as measured in the S-frame. The proton has velocity $\bar{u}'_p$ in the S'-frame. We want to transform $\bar{u}'_p$ to $\bar{u}_p$ in the S-frame.

The transformation equation is

$$u_p = \frac{u'_p + v}{1 + \frac{v u'_p}{c^2}}$$, plug in the velocities

$$u_p = \frac{0.70c + 0.90c}{1 + .7(.9)}$$

$$u_p = 0.98c$$

Note: the classical result would have been 1.6c.

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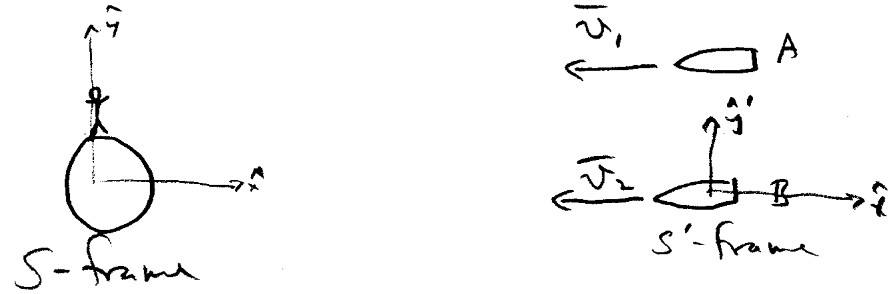
~~1-22~~



As measured in the S-frame $|\bar{v}_1| = .5c$ & $|\bar{v}_2| = .8c$.

We want to find the velocity of A as measured by B.

Classically, we'd say that B would measure the velocity of A to be

$$\bar{u}' = \bar{v}_1 - \bar{v}_2$$
$$= -0.5c\,\hat{x}' - (-0.8\,\hat{x}) = 0.3c\,\hat{x}'$$

i.e. $u_x' = 0.3c$

Relativistically, we use eqn 1.32, with $u_x = -v_1$ and $v = -v_2$

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-v_1 - (-v_2)}{1 - \left(\frac{v_1 v_2}{c^2}\right)}$$

$$u_x' = \frac{-.5c - (-.8c)}{1 - (.5(.8))} = \frac{.3c}{0.6} = 0.5c$$

Note the directions. The sign of u_x' depends on how the $\hat{x}$ & $\hat{x}'$ axes are oriented, etc.