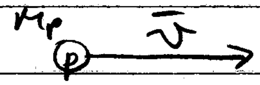
2-1 3rd ed

1-2



The mass of a proton is $m_p = 1.673 \times 10^{-27}$ kg.

The relativistic momentum is defined to be

$$|\bar{p}| = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m v.$$

	speed	p (kg m/sec)	p (MeV/c)
a)	.010c	5.01×10^{-21}	9.38
b)	0.50c	2.90×10^{-19}	542
c)	0.90c	1.04×10^{-18}	1930 (3 digits)

eg $v = 0.010c \rightarrow \gamma = 1.000 \rightarrow p = 1.673 \times 10^{-27}\ \text{kg}\ 3 \times 10^{6}\ \text{m/sec} =$

$p = 5.01 \times 10^{-21}\ \frac{\text{kg m}}{\text{sec}}$

In terms of $\frac{\text{MeV}}{c}$, $m_p = 938.3 \frac{\text{MeV}}{c^2}$, so

$$p = \gamma m_p (0.01c) = 9.38 \frac{\text{MeV}}{c}$$

For $v = 0.50c \rightarrow \gamma = 1.15$

For $v = 0.90c \rightarrow \gamma = 2.29$

2-2 3rd ed

~~1.27~~

Electron has $p_{rel} = \gamma m v = 1.9\, p_{clas} = 1.9\, m v$

a) $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m v = 1.9\, m v$ given

evidently

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.9 \quad \text{solve for } v$$

$$\left(\frac{1}{1.9}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$v^2 = .723\, c^2$$

$$v = 0.85c \quad (= 2.55 \times 10^8\ m/sec)$$

b) Wouldn't. The mv is common to both relativistic and classical momenta. So it divides out

2-9 3rd ed

a) The total energy is

$$E = \gamma \cdot mc^2 = K + mc^2$$

We're given that $K = 5mc^2$, so

$$E = 6mc^2$$ (look up mc^2 for electron)

$$E = 6\left(0.5117 \frac{MeV}{c^2}\right)c^2 =$$

$$E = 3.07 \text{ MeV} = 4.92 \times 10^{-13} \text{ J}$$

b) $\gamma = 6$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 6$$

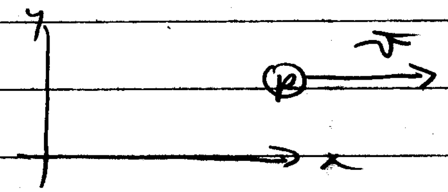
$$\frac{1}{36} = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{36}$$

$$v = 0.986c$$

2-11 3nd ed.

1-28



$m_p = 938.3\ MeV/c^2$

$K = 50\ GeV$

a) The energy, momentum and rest energy are related by

$$E^2 = p^2c^2 + m_p^2c^4$$

i) $E = K + m_p c^2 = 50\,GeV + 938.3\,MeV$

$= 50938.3\ MeV$

ii) Substitute this E into

$$E^2 = p^2c^2 + m_p^2c^4$$

$(50938.3\,MeV)^2 = p^2c^2 + (938.3\,MeV)^2$, solve for p

$$p^2c^2 = (50938.3\,MeV)^2 - (938.3\,MeV)^2$$

$$= 2.59 \times 10^9\ MeV^2$$

$p =$ [redacted] $\frac{MeV}{c}$ $\left(= 2.72 \times 10^{-17}\ \frac{kg\,m}{sec}\right)$

b) The total energy is also $E = \gamma m_p c^2$

$$50938.3\,MeV = \frac{938.3\,MeV}{\sqrt{1-\frac{v^2}{c^2}}}$$ solve for v

$v = 0.9998c$

3rd ed

2-11 in SI units

$m_p = 1.673 \times 10^{-27}\,kg \qquad m_p c^2 = 1.5057 \times 10^{-10}\,J$

a) $E = K + m_p c^2$

$E = 50\,GeV \cdot \dfrac{1.602 \times 10^{-19}\,J}{1\,eV} + 1.673 \times 10^{-27}\,kg \left(3 \times 10^{8} \frac{m}{sec}\right)^2$

$= 8.101 \times 10^{-9}\,J + 1.5057 \times 10^{-10}\,J$

$= 8.1606 \times 10^{-9}\,J$

$E^2 = p^2c^2 + m_p^2c^4$, solving for p

$p^2 = \frac{1}{c^2}\left(E^2 - m_p^2 c^4\right)$

$= \frac{1}{c^2}\left(\left(8.1606 \times 10^{-9}\,J\right)^2 - \left(1.5057 \times 10^{-10}\,J\right)^2\right)$

$= \frac{1}{c^2}\left[6.6572 \times 10^{-17}\,J^2\right] = 7.3969 \times 10^{-34}\,kg^2 \frac{m^2}{sec^2}$

$\left(\frac{sec}{m}\right)^2 \left(kg \frac{m^2}{sec^2}\right)^2$

$p = 2.7197 \times 10^{-17}\,kg \frac{m}{sec}$

2-11 cont.

b. $E = \gamma m_p c^2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_p c^2$

solving for v

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_p c^2}{E}$$

$$1-\frac{v^2}{c^2} = \frac{m_p^2 c^4}{E^2}$$

$$\frac{v^2}{c^2} = 1 + \frac{m_p^2 c^4}{E^2}$$

$$v^2 = c^2\left(1 - \frac{m_p^2 c^4}{E^2}\right)$$

$$v^2 = c^2\left[1 - \left(\frac{1.5057 \times 10^{-10} J}{8.1606 \times 10^{-9} J}\right)^2\right]$$

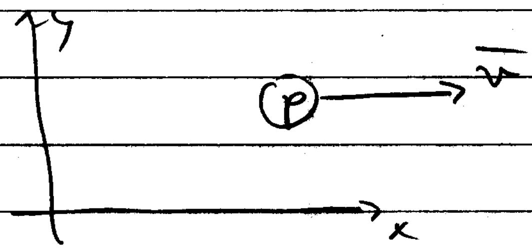
$$v^2 = c^2\left[1 - 3.4043 \times 10^{-4}\right]$$

$$v^2 = c^2 \, .99966$$

$$v = .9998\, c = 2.9995 \frac{m}{sec}$$

2-13 3rd ed

~~1-39~~



$m_p = 938.3 \frac{MeV}{c^2}$

$E = 400\, m_p c^2$

a) The total energy of the proton is

$E = 400\, m_p c^2$

Set this equal to $\gamma m_p c^2$

$$400\, m_p c^2 = \gamma m_p c^2$$

$$\gamma = 400$$, solve for v

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 400$$

$$v = 0.999997c$$

b) The kinetic energy is

$$K = \gamma m_p c^2 - m_p c^2$$

$$= (400-1)\, m_p c^2$$

$$= 399\, m_p c^2$$

$$= 3.74 \times 10^5 \text{ MeV}$$