3.23 3rd ed.

~~2.22~~ A photon has energy hf and momentum $\frac{h}{\lambda}$. Given $\lambda = 500nm$

$$E = hf = \frac{hc}{\lambda}$$

$$E = \frac{1240\,eV\,nm}{500\,nm}$$

$$E = 2.48\,eV \left(= 3.97 \times 10^{-19}\,J\right)$$

$$p = \frac{h}{\lambda} = \frac{E}{c}$$

$$p = \frac{3.97 \times 10^{-19}\,J}{3 \times 10^{8}\,m/sec}$$

$$= 1.32 \times 10^{-27}\,\frac{kg\,m}{sec} = 2.48\,\frac{eV}{c}$$

3-24 3rd ed

~~2.23~~ Here we have Compton scattering of photons from the electrons in a block of carbon.

The Compton Shift is given by

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

We're given that $\lambda_0 = 0.200\,nm$ and $\theta = 90^\circ$

a) $$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

$$\Delta\lambda = \frac{6.625\times10^{-34}\,J\,sec}{9.1\times10^{-31}\,kg\;\;3\times10^{8}\,\frac{m}{sec}}(1-\cos 90^\circ)$$

$$\Delta\lambda = 2.43\times10^{-12}\,m = 2.43\times10^{-3}\,nm$$

b) Energy is conserved so the energy lost by the photon is imparted to the electron

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda} + K_e \quad , \text{ solve for } K_e$$

$$K_e = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)$$

$$= 1240\,eV\cdot nm\left(\frac{1}{.200\,nm} - \frac{1}{.20243\,nm}\right)$$

$$= 74.4\,eV$$

3.25 3rd ed

~~2.24~~ Compton Scattering

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

In this case, we're given the energy of the incident photons: $E = \frac{hc}{\lambda_0} = 300\,keV$.

The scattering angle is $\theta = 30°$.

a) The Compton shift is

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

$$= \frac{6.63\times10^{-34}\,J\,sec}{9.11\times10^{-31}\,kg\;3\times10^{8}\,\frac{m}{sec}}(1-\cos 30°)$$

$$= 3.25\times10^{-13}\,m = 3.25\times10^{-4}\,nm$$

b) The wavelength of the scattered photon is

$$\lambda = \lambda_0 + \Delta\lambda = .00413\,nm + 3.25\times10^{-4}\,nm$$

$$= .00446\,nm$$

Its energy is $E = \frac{hc}{\lambda} = 278\,keV$ $(4.45\times10^{-14}\,J)$

c) The kinetic energy passed to the electron is

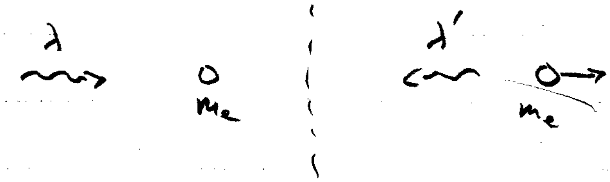
$$K = 300\,keV - 278\,keV = 22\,keV.$$

$(3.52\times10^{-15}\,J)$

2-30 3rd ed

~~2-30~~

$$E = E' + K_e$$



$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + K_e$$

$$K_e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{hc(\lambda'-\lambda)}{\lambda\lambda'} = \frac{hc\Delta\lambda}{\lambda\lambda'}.$$

The maximum energy transfer occurs when $\Delta\lambda$ is a maximum.

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\cancel{\theta}^{\,\pi}) = \frac{2h}{m_e c}.$$

Substitute this into K_e (classical)

$$K_e = \frac{hc}{\lambda\lambda'}\,\frac{2h}{m_e c} = \frac{2h^2}{m_e\lambda\lambda'} = 30\,keV$$

Now, we see that $\Delta\lambda$ is small, so let $\lambda' \simeq \lambda$

$$K_e \simeq \frac{2h^2}{m_e\lambda^2}\text{ ; solve for } \lambda$$

$$\lambda \simeq \sqrt{\frac{2h^2}{m_e K_e}} = \sqrt{\frac{2(4.14\times10^{-15}\,eV\cdot sec)^2}{0.511\,\frac{MeV}{c^2}\;30\,keV}}$$

$$\lambda \simeq 1.42\times10^{-11}\,m = 1.42\times10^{-2}\,nm$$

(don't forget the c^2!)

3-30 3rd ed.

~~2-30~~ the "right" way:

$$\Delta E = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right)$$

$$\lambda' - \lambda = 2\frac{h}{m_e c}$$

$$|\Delta E| = 30\,keV \cdot \frac{1.602\times10^{-19}\,J}{1\,eV} = 4.806\times10^{-15}\,J$$

$$\lambda' = \frac{2h}{m_e c} + \lambda$$

$$\Delta E = hc\left(\frac{1}{\frac{2h}{m_e c} + \lambda} - \frac{1}{\lambda}\right)$$

$$\Delta E = hc\left(\frac{\lambda - \frac{2h}{m_e c} - \lambda}{\left(\frac{2h}{m_e c} + \lambda\right)\lambda}\right)$$

$$\Delta E = \frac{hc\left(-\frac{2h}{m_e c}\right)}{\frac{2h}{m_e c}\lambda + \lambda^2} = -\frac{\frac{2h^2}{m_e}}{\frac{2h}{m_e c}\lambda + \lambda^2}$$

$$\frac{2h}{m_e c}\lambda + \lambda^2 = -\frac{2h^2}{m_e \Delta E}$$

Note $\Delta E < 0$ – the photon loses energy.

$$\lambda^2 + \frac{2h}{m_e c}\lambda + \frac{2h^2}{m_e \Delta E} = 0$$

$$\lambda^2 + 4.85\times10^{-12}\,m\;\lambda + \left(-2.00\times10^{-22}\,m^2\right) = 0$$

$ax^2 + bx + c = 0$

$$\lambda = \frac{-4.85\times10^{-12}\,m \pm \sqrt{2.352\times10^{-23}\,m^2 - 4\cdot1\cdot(-2\times10^{-22}\,m^2)}}{2}$$

$$\lambda = \frac{-4.85\times10^{-12}\,m + 2.87\times10^{-11}\,m}{2} = 1.19\times10^{-4}\,m = 1.2\times10^{-2}\,nm$$

~~3-43~~ 3-43 2nd ed

~~2-37~~

Compton Effect

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

Here, $\lambda_0 = 4000\,Å = 4\times10^{-7}\,m$

$\theta = 180°$

$$\lambda' = \lambda_0 + \frac{2h}{m_e c} = 4000.04\,Å$$

a) The energy transferred to the electron is the energy lost by the photon(s)

$$\Delta E = \frac{hc}{\lambda_0} - \frac{hc}{\lambda'}$$ (don't round off!!)

$$\Delta E = 1.989\times10^{-25}\,J\,m\ [25\,m^{-1}]$$

$$\Delta E = 4.97\times10^{-24}\,J = 3\times10^{-5}\,eV$$

b) In the photoelectric effect

$$\Delta E = K_{max} = hf_0 - \phi$$

$$= \frac{hc}{\lambda_0} - \phi$$

$$= 4.97\times10^{-19}\,J - \phi$$

$$K_{max} = 3.10\,eV - \phi$$

$\phi \approx$ few eV, so $K_{max} \approx 0$.

c) Violet light ~~bec~~ could not eject a photon from a metal via the Compton effect, because $\Delta E \ll$ the typical ϕ.