5.2 – 3rd ed

~~4.1~~ The deBroglie wavelength of a particle is $\lambda = \frac{h}{p}$, where p is the particle momentum and h is Planck's Constant

The momentum is related to the kinetic energy thusly $K = \frac{p^2}{2m}$ or $p = \sqrt{2mK}$. (nonrelativistic)

a) The electron mass is 9.1095×10^{-31} kg.

If $K = 50\,eV = 8.01 \times 10^{-18}$ J, then

$$p = \sqrt{2(9.1095 \times 10^{-31}\,kg)\,8.01 \times 10^{-18}\,J}$$

$$p = 3.82 \times 10^{-24}\ kg\,\frac{m}{sec}$$

Thus, $\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ J \cdot sec}{3.82 \times 10^{-24}\ kg\,\frac{m}{sec}}$

$$\lambda = 1.73 \times 10^{-10}\ m$$

b) If $K = 50\ keV = 8.01 \times 10^{-15}$ J, then

$p = 1.21 \times 10^{-22}\ kg\frac{m}{sec}$ and

$$\lambda = 5.48 \times 10^{-12}\ m$$

Note: One could have left the K in eV, and used $h = 4.136 \times 10^{-15}\ eV \cdot sec$

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~~4-2~~ (relativistic)

deBroglie wavelength $\lambda = \frac{h}{p}$

The kinetic energy of a particle is

$K = E - mc^2$ where $E = \gamma mc^2$.

a) We've given $K = 50\,eV$. Solve for E
and $m = .511\,\frac{MeV}{c^2}$

$E = K + mc^2 = 50\,eV + .511\,\frac{MeV}{c^2} \cdot c^2$
$= 511050\,eV$

We also have
$E^2 = p^2c^2 + m^2c^4$. Solve for p

$p^2c^2 = E^2 - m^2c^4$
$= (511050\,eV)^2 - (.511\,MeV)^2$
$= 51102500\,eV^2$

$p = 7,150\,\frac{eV}{c}$.

Finally
$\lambda = \frac{h}{p} = \frac{4,136 \times 10^{-15}\,eV\,sec}{7150\,\frac{eV}{c}}$

$\lambda = 1,74\,\text{Å}$

Ditto for part b) with $K = 50 \times 10^3\,eV$.

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~~4-3~~ $m = 74\,kg \quad v = 5.0\,m/sec$

deBroglie wavelength $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}\,J \cdot sec}{74\,kg \;\; 5.0\,m/sec}$$

$$= 1.79 \times 10^{-36}\,m = 1.79 \times 10^{-26}\,\mathring{A}$$

5-4 3rd ed

~~4.4~~ We want to know, for what momentum, p, will an electron's deBroglie wavelength be $\lambda = 0.1 \times 10^{-9}$m.

Well, $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$

$$p = \frac{6.626 \times 10^{-34}\, J \cdot sec}{0.1 \times 10^{-9} m} = 6.626 \times 10^{-24}\, \frac{kg\, m}{sec}$$

Set $p = mv$, and solving for v

$$v = \frac{p}{m} = \frac{6.626 \times 10^{-24}\, kg\, m/sec}{9.1095 \times 10^{-31} kg}$$

$$v = 7.27 \times 10^{6}\, \frac{m}{sec}$$

Note: This speed is just $\simeq 0.024c$, so we didn't need to use relativistic expressions.

5-5 3rd ed

~~4-5~~

$$K = \gamma mc^2 - mc^2 = E - mc^2$$

$$p = \frac{h}{\lambda} = \gamma m u$$

Given λ, we want to find p and then K, the kinetic energy

The relativistic equation relating p and E is

$$E^2 = p^2c^2 + m^2c^4 .$$ Plug in $p = \frac{h}{\lambda}$

$$E^2 = \frac{h^2}{\lambda^2}c^2 + m^2c^4 .$$ Solve for E,

plug into the equation for K

$$K = \sqrt{\frac{h^2}{\lambda^2}c^2 + m^2c^4} - mc^2 .$$

For an electron $mc^2 = .5109991$ MeV

For an α-particle $mc^2 \cong 4uc^2 = 4(931.5 \text{ MeV}) = 3726 \text{ MeV}$

$$h = 6.626076 \times 10^{-34} \text{ J}\cdot\text{sec} \cdot \frac{1 \text{ eV}}{1.6021773 \times 10^{-19} \text{ J}} = 4.13567 \times 10^{-15} \text{ eV sec}$$

$$c = 2.99792458 \times 10^{8} \frac{\text{m}}{\text{sec}}$$

Plug in and turn the crank...

3^{rd} ed

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~~4-5~~

	mc^2 MeV	λ (m)	—	K (eV)	
a,	.511	10×10^{-9}		1.50×10^{-2}	
b.	.511	$.1\times10^{-9}$		1.50×10^{2}	
c.	.511	10×10^{-15}		$1.23\times10^{+8}$	
a.	3726	10×10^{-9}		0 !	(2.06×10^{-6})
b.	3726	$.1\times10^{-9}$		2.06×10^{-2}	
c.	3726	10×10^{-15}		2.06×10^{6}	

This is an example of a situation wherein we need to plug in the constants with as many significant digits as we can find.

We may use the nonrelativistic $\lambda = \frac{h}{\sqrt{2mK}}$, but at the shortest wavelength, the result is a little off.