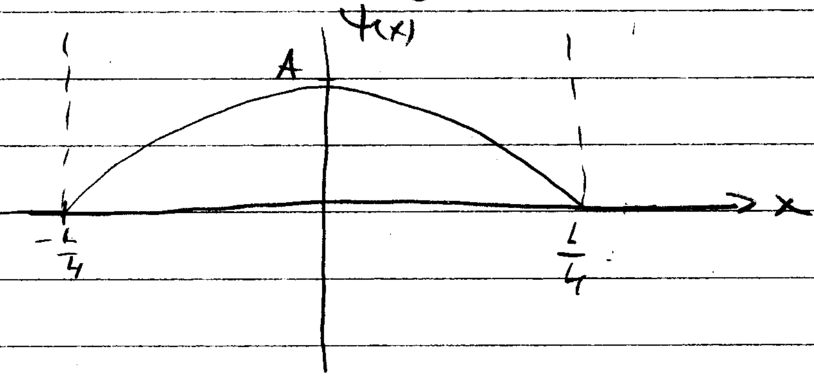
6.2 3^{rd} ed

~~5.2~~

$$\psi(x) = \begin{cases} A\cos\frac{2\pi x}{L} & -\frac{L}{4} \le x \le \frac{L}{4} \\ 0 & \text{otherwise} \end{cases}$$



a) We require that

$$\int_{-\frac{L}{4}}^{\frac{L}{4}} |\psi(x)|^2 dx = 1$$

$$\left[\int \cos^2 ax\, dx = \frac{1}{2}x + \frac{1}{4a}\sin 2ax \right]$$

$$\int_{-\frac{L}{4}}^{L/4} A^2 \cos^2 \frac{2\pi x}{L}\, dx = 1$$

$$A^2 \frac{L}{4} = 1$$

$$A = \sqrt{\frac{4}{L}}$$

b) The probability that the particle is found in the interval $0 \le x \le \frac{L}{8}$ is

$$P = \int_0^{L/8} \left(\sqrt{\frac{4}{L}} \cos \frac{2\pi x}{L} \right)^2 dx$$

$$= \frac{4}{L}\left[\frac{L}{16} + \frac{L}{8\pi} \sin\frac{\pi}{2} \right] = 0.409$$

6.6 3rd ed.

5.6 $\psi(x) = A\cos kx + B\sin kx$

For a free particle $U(x) = 0$, so the Schrödinger eqn becomes just

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi.$$

Plug in the given ψ

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left(A\cos kx + B\sin kx\right) = E\left(A\cos kx + B\sin kx\right)$$

Take the derivatives:

$$\frac{d^2}{dx^2} A\cos kx = -Ak^2\cos kx$$

$$\frac{d^2}{dx^2} B\sin kx = -Bk^2\sin kx.$$

So, equating

$$-\frac{\hbar^2}{2m}(-k^2)\left[A\cos kx + B\sin kx\right] = E\left(A\cos kx + B\sin kx\right)$$

$$+\frac{\hbar^2 k^2}{2m}\psi = E\psi$$

Evidently, $E = \dfrac{\hbar^2 k^2}{2m}$.

6-9 3rd ed

~~5.9~~ Again, the energy levels for a particle confined in an infinitely deep box are

$$E_n = \frac{n^2 h^2}{8mL^2} \quad \text{or} \quad \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

We're given $L = 10^{-5}\,\text{nm} = 10^{-14}\,\text{m}$ and

$$m = m_p = 1.673 \times 10^{-27}\,\text{kg}.$$

The energy change in a transition from $n=2$ to $n=1$ states is

$$\Delta E = \frac{h^2}{8m_pL^2}\left(2^2 - 1^2\right) = 9.84 \times 10^{-13}\,\text{J}$$

$4-1=3$ $\qquad$ $6.14\,\text{MeV}$

The wavelength emitted is $\lambda = \dfrac{hc}{\Delta E}$

$$\lambda = \frac{6.623 \times 10^{-34}\,\text{J}\cdot\text{sec} \cdot 3 \times 10^{8}\,\text{m/sec}}{9.84 \times 10^{-13}\,\text{J}}$$

$$\lambda = 2.02 \times 10^{-13}\,\text{m}$$

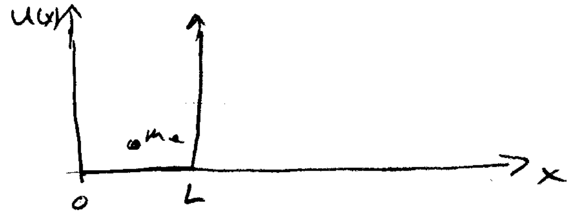
This is in the γ-ray portion of the spectrum.

6-10 3rd ed

~~5~~ 10

$L = 0.1\,nm$

$m_e = 9.109 \times 10^{-31} kg = .511 \frac{MeV}{c^2}$



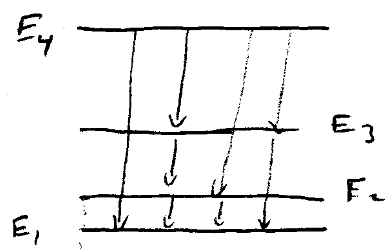
!

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2 m_e L^2} = n^2 \, 4.184 \times 10^{-16} \, eV \frac{sec^2}{m^2} \cdot c^2$$

$E_n = n^2 \, 37.65\,eV$

[don't forget the c^2 in m_e if you use $m_e = .511 \frac{MeV}{c^2}$]
[Check the units]

a) $E_1 = 37.65\,eV$
$E_2 = 150.61\,eV$
$E_3 = 338.85\,eV$
$E_4 = 602.40\,eV$



see fig ~~5.7~~ 6.7

$hf = \Delta E$

$\frac{hc}{\lambda}$

b) There are 4 different sequences of transition from $n=4$ to $n=1$. But, some sequences involve the same photon energies.

We just need

$\Delta E = E_4 - E_1 = 564.75\,eV \Rightarrow \lambda = 2.2 \times 10^{-9}\,m$

$\Delta E = E_3 - E_1 = 301.2\,eV \Rightarrow \lambda = 4.1 \times 10^{-9}\,m$

$\lambda = \frac{hc}{\Delta E} = \frac{4.136 \times 10^{-15}\,eV\,sec \cdot c}{\Delta E}$

$\Delta E = E_2 - E_1 = 112.96\,eV \Rightarrow \lambda = 1.1 \times 10^{-9}\,m$

$\Delta E = E_4 - E_2 = 451.79\,eV \Rightarrow \lambda = 2.7 \times 10^{-9}\,m$

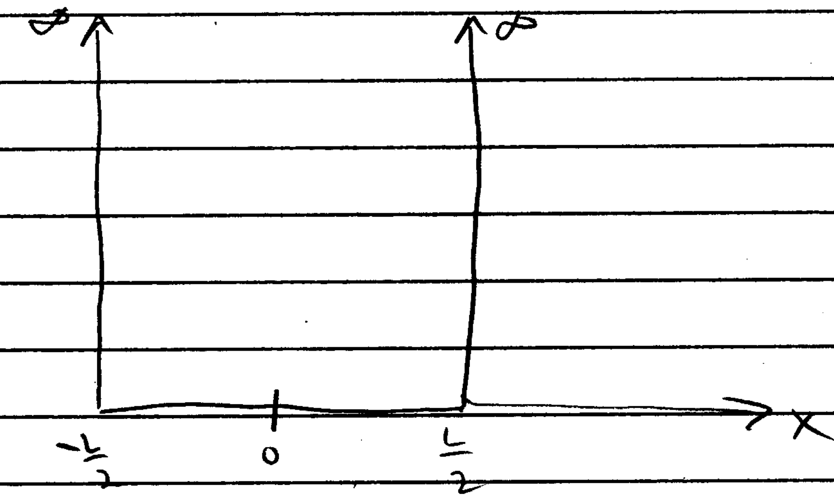
$\Delta E = E_3 - E_2 = 188.24\,eV \Rightarrow \lambda = 6.6 \times 10^{-9}\,m$

$\Delta E = E_4 - E_3 = 263.55\,eV \Rightarrow \lambda = 4.7 \times 10^{-9}\,m$

graded.

6-11

a)



The simplest thing to do is to use the solutions for the box centered at $\frac{L}{2}$, and just shift the origin.

$$\sin\left(\frac{n\pi}{L}x\right) \longrightarrow \sin\left(\frac{n\pi}{L}\left(x-\frac{L}{2}\right)\right)$$

$$\sin\left(\frac{n\pi}{L}x - \frac{n\pi}{2}\right)$$

Use the identity for the sine of the difference of 2 angles

$$\sin\left(\frac{n\pi x}{L} - \frac{n\pi}{2}\right) = \sin\frac{n\pi x}{L}\cos\frac{n\pi}{2} - \cos\frac{n\pi x}{L}\sin\frac{n\pi}{2}$$

Now, $\cos\frac{n\pi}{2} = 0$ and $\sin\frac{n\pi}{2} = \pm 1$ when n is odd

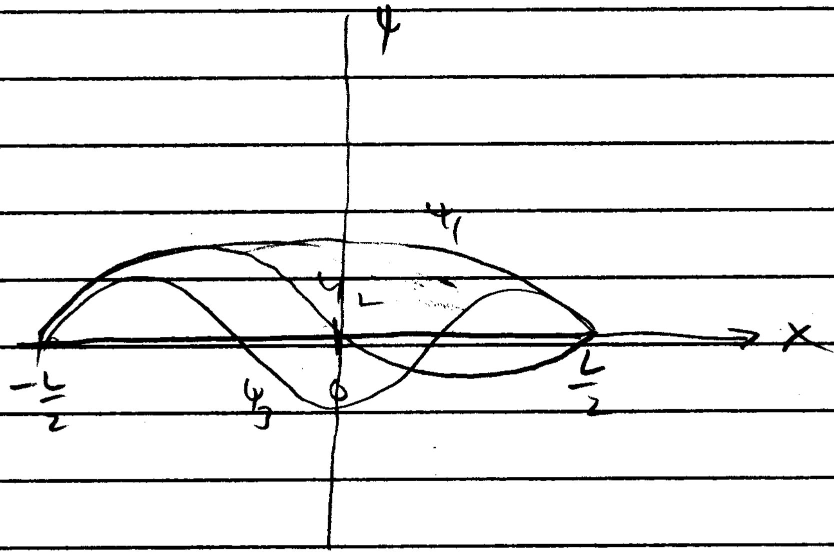
$= \pm 1$ $\qquad$ 0 when n is even.

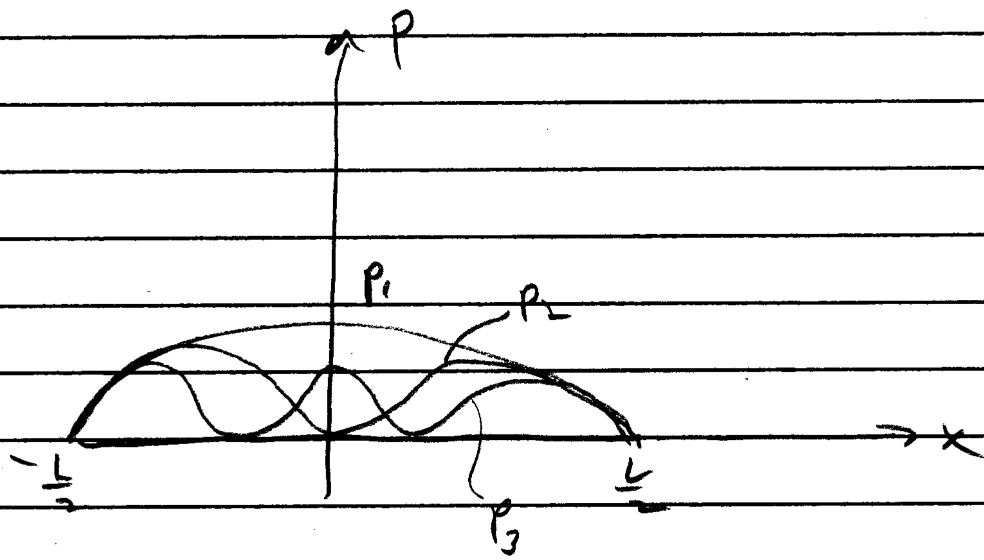
$\psi_1(x) = \sqrt{\frac{2}{L}}\cos\frac{\pi x}{L}$; $P_1(x) = \frac{2}{L}\cos^2\frac{\pi x}{L}$

$\psi_2 = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$; $P_2(x) = \frac{2}{L}\sin^2\frac{2\pi x}{L}$

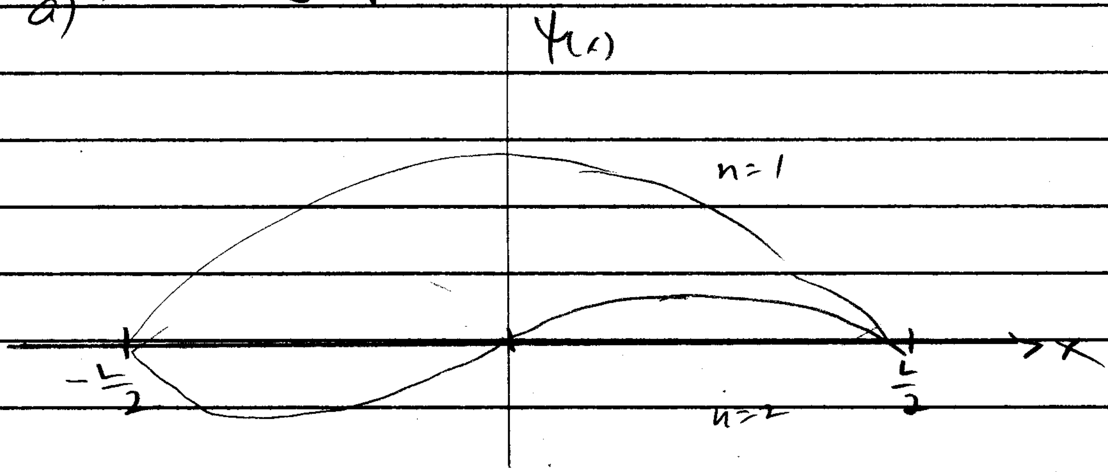
$\psi_3 = \sqrt{\frac{2}{L}}\cos\frac{3\pi x}{L}$; $P_3(x) = \frac{2}{L}\cos^2\frac{3\pi x}{L}$ etc

6-11 b





6-11 a) from scratch



$$\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi \quad \text{for} \quad -\frac{L}{2} < x < \frac{L}{2}$$

let $\psi = A\sin\frac{n\pi}{L}x + B\cos\frac{n\pi}{L}x$

The boundary conditions are

$$\psi\left(x=-\frac{L}{2}\right) = 0 \quad \text{and} \quad \psi\left(x=+\frac{L}{2}\right) = 0$$

$$A\sin\left(-\frac{n\pi L}{L\,2}\right) + B\cos\left(-\frac{n\pi L}{2L}\right) = 0$$

$$A\sin\frac{n\pi L}{2L} + B\cos\frac{n\pi L}{2L} = 0$$

add 'em $\quad 2B\cos\frac{n\pi}{2} = 0$

If n is even, then $B = 0$ since $\cos\frac{n\pi}{2} = \pm 1$.

subtract 'em $\quad 2A\sin\frac{n\pi}{2} = 0$

If n is odd, then $A = 0$ since $\sin\frac{n\pi}{2} = \pm 1$

So we have $\psi_n = B\cos\frac{n\pi x}{L}$ for odd n

$\psi_n = A\sin\frac{n\pi x}{L}$ for even n.

When normalized, $A = B = \sqrt{\frac{2}{L}}$.

3rd ed

6-12

$\lambda = 694.3$ nm

The energy levels for an electron in a box are

$$E_n = n^2 \frac{h^2}{8mL^2}$$

$L = n = 2$ and 1 ; $m = 9.109 \times 10^{-31}$ kg $= .511 \frac{MeV}{c^2}$

$$\Delta E = E_2 - E_1 = (4-1)\frac{h^2}{8mL^2}$$

also

$$\Delta E = \frac{hc}{\lambda}$$

$$(4-1)\frac{h^{\cancel{2}}}{8mL^2} = \frac{\cancel{h}c}{\lambda}$$, solve for L

$$L = \sqrt{\frac{\lambda h}{8mc}3} = \sqrt{\frac{(3)\, 694.3 \times 10^{-9}\, m \;\; 4.136 \times 10^{-15}\, eVsec}{(8)\; .511 \times 10^{6} \frac{eV}{c}}}$$

$$L = 7.95 \times 10^{-10}\, m = 7.95\, Å$$

6-13 3rd ed

~~5-13~~ Yet another particle confined in a one-dimensional square well.

The energy levels are $E_n = \frac{n^2 h^2}{8mL^2}$

a) In this case

$L = 0.200\ nm = 2 \times 10^{-10}\ m$

$m = m_p = 1.67 \times 10^{-27}\ kg = 938.3\ MeV/c^2$

with $n = 1$, the ground state energy is

$E_1 = \frac{h^2}{8m_p L^2} = 8.22 \times 10^{-22}\ J = 5.13 \times 10^{-3}\ eV$

b) If an electron is confined in the box, rather than a proton, what's different is $\underline{m = m_e}$ ... $[m_e = 9.11 \times 10^{-31}\ kg]$

$E_1 = \frac{h^2}{8m_e L^2} = 1.51 \times 10^{-18}\ J \approx 9.40\ eV$

c) $m_p \approx 2000\ m_e$

3^{rd} ed

6-24

$$\psi(x) = Cx\,e^{-\alpha x^2}$$

a) $$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + U(x)\,\psi = E\,\psi$$

with $U(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$ — ω = classical angular frequency

$$\frac{d^2}{dx^2}\psi = \frac{2m}{\hbar^2}\left(U(x) - E\right)\psi$$

$$\frac{d}{dx}\left(Cxe^{-\alpha x^2}\right) = -2\alpha x\,Cxe^{-\alpha x^2} + Ce^{-\alpha x^2}$$

$$= -2\alpha x\,\psi(x) + Ce^{-\alpha x^2}$$

$$\frac{d^2}{dx^2}\left(Cxe^{-\alpha x^2}\right) = -2\alpha x\frac{d\psi}{dx} - 2\alpha\psi(x) - (2\alpha x)Ce^{-\alpha x^2}$$

$$= (2\alpha x)^2\psi(x) - 6\alpha\psi(x)$$

$$(2\alpha x)^2\cancel{\psi(x)} - 6\alpha\,\cancel{\psi(x)} = \frac{2m}{\hbar^2}\left(\frac{1}{2}m\omega^2x^2 - E\right)\cancel{\psi(x)}$$

$$(2\alpha x)^2 - 6\alpha = \left(\frac{m\omega}{\hbar}\right)^2x^2 - \frac{2mE}{\hbar^2}$$

equate coefficients of like powers of x

$$2\alpha = \frac{m\omega}{\hbar} \quad \text{and} \quad 6\alpha = \frac{2mE}{\hbar^2}$$

$$\alpha = \frac{m\omega}{2\hbar} \quad \text{and} \quad E = \frac{6\alpha\hbar^2}{2m} = \frac{3}{2}\hbar\omega$$

6-24

b)

$$\int_{-\infty}^{\infty} |\psi|^2\, dx = 1$$

$$\int_{-\infty}^{\infty} C^2 x^2 e^{-2\alpha x^2}\, dx = 1$$

$$C^2 \int_{-\infty}^{\infty} x^2 e^{-2\alpha x^2}\, dx = 1$$

$$2C^2 \int_0^{\infty} x^2 e^{-2\alpha x^2}\, dx = 1$$

$$2C^2 \left(\frac{1}{8\alpha}\right)\left(\frac{\pi}{2\alpha}\right)^{1/2} = 1$$

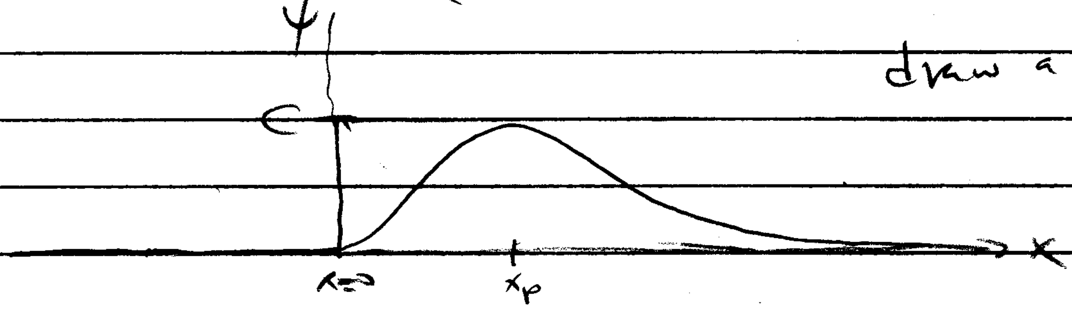
$$C = \left(\frac{32\alpha^3}{\pi}\right)^{\frac{1}{4}}.$$

$$= \left[\frac{4}{\pi}\left(\frac{m\omega}{\hbar}\right)^3\right]^{1/4}$$

6-29

$$\psi(x) = \begin{cases} 0 & x < 0 \\ C e^{-x}(1-e^{-x}) & x > 0 \end{cases}$$

draw a picture!



a) $$\int_0^\infty |\psi|^2 \, dx = 1$$

$$\int_0^\infty C^2 e^{-2x} (1-e^{-x})^2 \, dx = 1$$

$$C^2 \int_0^\infty \left(e^{-2x} - 2e^{-3x} + e^{-4x}\right) dx = 1$$

$$C^2 \left(\frac{1}{2} - 2\left(\frac{1}{3}\right) + \frac{1}{4} \right) = 1$$

$$C^2 = 12$$

b) $$P(x)\,dx = |\psi|^2 dx = C^2 e^{-2x}(1-e^{-x})^2 \, dx$$
$$= C^2 (e^{-2x} - 2e^{-3x} + e^{-4x}) \, dx$$

set $$\frac{dP}{dx} = 0$$

$$C^2(-2e^{-2x} + 6e^{-3x} - 4e^{-4x}) = 0$$

6-29

$\frac{dP}{dx} = 0$ when $x \to \infty$ and when

$$2e^{-x} - 1 = 0$$

i.e. when $x_p = \ln 2 = .693$.

c) $$\langle x \rangle = \int_0^\infty x |\psi|^2 \, dx$$

$$= C^2 \int_0^\infty x e^{-2x} (1-e^{-x})^2 \, dx$$

$$= C^2 \int_0^\infty x \left(e^{-2x} - 2e^{-3x} + e^{-4x}\right) dx$$

$$= C^2 \left\{ \frac{1}{4} - 2\left(\frac{1}{9}\right) + \frac{1}{16} \right\}$$

$$= C^2 \, \frac{13}{144}$$

$$\langle x \rangle = 1.083$$

$\langle x \rangle > x_p$, since the $|\psi|^2$ is not symmetric.

6-30

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$\langle x \rangle = \frac{2}{L}\int_0^L x \sin^2 \frac{n\pi x}{L}\, dx$$

I'd look it up

$$\langle x \rangle = \frac{2}{L}\left[\frac{x^2}{4} - \frac{x \sin \frac{2n\pi x}{L}}{4\frac{n\pi}{L}} - \frac{\cos \frac{2n\pi x}{L}}{8\left(\frac{n\pi}{L}\right)^2} \right]_0^L$$

$$= \frac{L}{2}$$

$$\langle x^2 \rangle = \frac{2}{L}\int_0^L x^2 \sin^2 \frac{n\pi x}{L}\, dx$$

$$= \frac{2}{L}\left[\frac{x^3}{6} - \left(\frac{x^2}{4\frac{n\pi}{L}} - \frac{1}{8\left(\frac{n\pi}{L}\right)^3} \right) \sin \frac{2n\pi x}{L} - \frac{x \cos \frac{2n\pi x}{L}}{4\left(\frac{n\pi}{L}\right)^2} \right]_0^L$$

time passes ⋮

$$= \frac{L^2}{3} - \frac{L^2}{2(n\pi)^2}$$