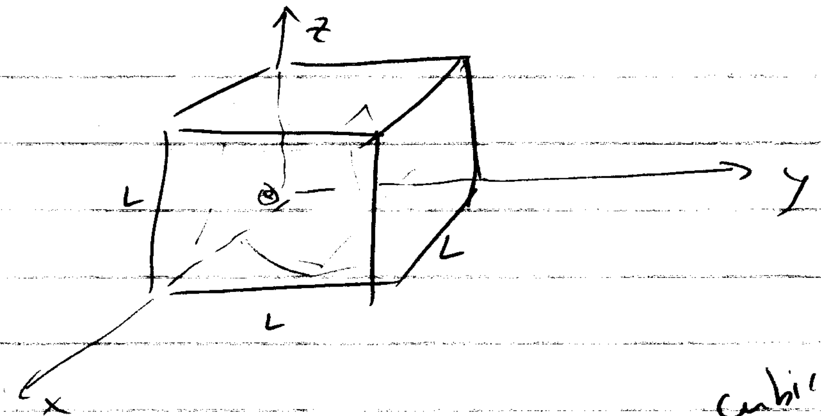
~~7/2~~ 8-2 3rd ed



The electron is confined to a cubical box of side $L = 0.2 \times 10^{-9}$ m. The energy levels are

$$E = \frac{\pi^2 \hbar^2}{2m L^2} (n_1^2 + n_2^2 + n_3^2)$$

$$= 9.41 \text{ eV} (n_1^2 + n_2^2 + n_3^2)$$

a) The ground state energy occurs when $n_1 = n_2 = n_3 = 1$, whence

$$E_{111} = 3(9.41 \text{ eV}) = 28.2 \text{ eV} = 4.52 \times 10^{-18} \text{ J}$$

b.) The first excited state occurs when $n_1 = 2$ or $n_2 = 2$ or $n_3 = 2$

$$E_{211} = E_{121} = E_{112} = 6(9.41 \text{ eV}) = 56.5 \text{ eV}.$$

$$= 9.05 \times 10^{-18} \text{ J}$$

3^{rd} ed

8-10

~~7~~-10 An electron is in a state
$n=4$, $l=3$ and $m_l=3$.

a) $|\bar{L}|^2 = l(l+1)\hbar^2$
$= 12\hbar^2 \Rightarrow |\bar{L}| = 3.65\times10^{-34}\ \frac{kg\,m^2}{sec}$
$= 2.28\times10^{-15}\ eV\,sec$

b) $L_z = m_l\hbar = 3\hbar = 3.17\times10^{-34}\ \frac{kg\,m^2}{sec}$
$= 1.97\times10^{-15}\ eV\,sec$

8-11 3rd ed

~~7-11~~ If the orbital angular momentum of the Earth is quantized (which it is), what is the value of its orbital quantum number, ℓ? Given $L = 4.83 \times 10^{31}\,kg\,\frac{m^2}{sec}$, and $L^2 = \ell(\ell+1)\hbar^2$.

a) $$\ell(\ell+1)\hbar^2 = \left(4.83\times10^{31}\,kg\,\frac{m^2}{sec}\right)^2$$

$$\ell(\ell+1) = \frac{\left(4.83\times10^{31}\,\cancel{kg\,\frac{m^2}{sec}}\right)^2}{\left(1.055\times10^{-34}\,\cancel{J\,sec}\right)^2} \quad \text{(units cancel)}$$

$$= \left(4.85\times10^{65}\right)^2$$

$$\ell^2 + \ell = 23.5\times10^{130}$$

approximately, $\ell \cong \sqrt{23.5\times10^{130}} \approx 4.85\times10^{65}$

b) If $\ell \to \ell+1$, the $\Delta L = \left[\sqrt{(\ell+1)(\ell+2)} - \sqrt{\ell(\ell+1)}\right]\hbar$

The fractional change is $\frac{\Delta L}{L} = \frac{\Delta L}{\sqrt{\ell(\ell+1)}\,\hbar}$.

So, $$\frac{\Delta L}{L} = \sqrt{\frac{(\ell+1)(\ell+2)}{\ell(\ell+1)}} - 1 \approx 0$$

Too small to ever detect.
I mean, too small ever to detect.
Sophisticated persons do not split infinitives.

8-22

$$R_{2p}(r) = A r e^{-r/2a_0}$$

Expectation value of r $\int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} r|\psi|^2 r^2 \sin\theta \, d\theta \, d\phi \, dr$

$$\langle r \rangle = 4\pi \int_0^{\infty} r\left(A^2 r^2 e^{-\frac{2r}{2a_0}}\right) r^2 \, dr$$

$$= 4\pi A^2 \int_0^{\infty} r^5 e^{-\frac{r}{a_0}} \, dr$$

Look up $\int_0^{\infty} x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}}$

$$\langle r \rangle = 4\pi A^2 \frac{5!}{\left(\frac{1}{a_0}\right)^6}$$

We need A^2: $4\pi \int_0^{\infty} A^2 r^2 e^{-\frac{2r}{2a_0}} r^2 \, dr = 1$

$$4\pi A^2 \frac{4!}{\left(\frac{1}{a_0}\right)^5} = 1$$

$$A^2 = \frac{1}{4\pi (4!) a_0^5}$$

$$\langle r \rangle = \frac{4\pi \; 5! \; a_0^6}{4\pi \, (4!) \, a_0^5} = 5 a_0 = 2.65 \times 10^{-10} \text{ m} = 2.65 \text{ Å}$$

check against Fig 8.11 ...