3.10 $m = 10\,kg$ $k = 250\,N/m$ $c = 60\,\frac{kg}{sec}$

$F_d = F_0 \cos\omega t$, $F_0 = 48\,N.$

a) The steady state solution to the driven, damped harmonic oscillator occurs at resonance, $\omega = \omega_r$

$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$ eqn 3.6.10

$$= \omega_0^2 - 2\left(\frac{c}{2m}\right)^2$$ eqn 3.4.3

$$= \omega_0^2 - \frac{c^2}{2m^2} = \frac{k}{m} - \frac{c^2}{2m^2}$$

$$= 25\,sec^{-2} - 18\,sec^{-2} = 7\,sec^{-2}$$

$$\omega_r = 2.646\,sec^{-1}$$

b) at $\omega = \omega_r$

$$A(\omega) = \frac{F_0/m}{\left[(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2\right]^{1/2}}$$

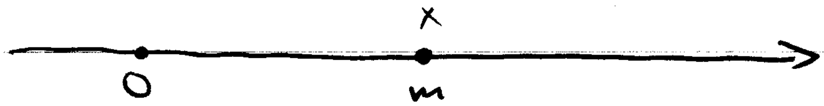
$\gamma = \frac{c}{2m} = 3\,sec^{-1}$ $\omega_0 = \sqrt{\frac{k}{m}} = 5.0\,sec^{-1}$ $\omega = \omega_r = 2.646\,sec^{-1}$

$$A(\omega_r) = \frac{48N/10kg}{\left[(5.0^2 - 7)^2 + 4(9)7\right]^{1/2}}$$

$$A(\omega_r) = \cancel{0.0??}\ 0.2\ m$$

c) $\tan\phi = \frac{c\omega_r}{k - m\omega_r^2} = 0.882$; $\phi = 41.4°$ eqn 3.6.7c

3.11



$$m\ddot{x} = -kx - c\dot{x} + F_0\cos\omega t$$

$$m\ddot{x} = -17\beta^2\frac{m}{2}x - 3\beta m\dot{x} + mA\cos\omega t$$

Indentify $k = 17\beta^2\frac{m}{2}$, $c = 3\beta m$, $F_0 = mA$

$$\omega_0 = \sqrt{\frac{k}{m}} = 2.92\beta \;; \qquad \gamma = \frac{c}{2m} = \frac{3\beta m}{2m} = \frac{3}{2}\beta$$

a. Resonant driving frequency is $\omega = \omega_r$

$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$

$$\omega_r^2 = \left(\frac{17}{2} - \frac{9}{2}\right)\beta^2$$

$$\omega_r^2 = 4\beta^2$$

b. $$\frac{F_0/m}{2\gamma\sqrt{\omega_0^2-\gamma^2}} = \frac{A}{2(\frac{3}{2}\beta)\sqrt{\frac{17}{2}\beta^2 - (\frac{3}{2}\beta)^2}}$$

$$A(\omega_r) = \frac{A}{3\beta\ 2.5\beta}$$

$$A(\omega_r) = \frac{A}{7.5\beta^2}$$

3.14

Damped oscillator

$$x(t) = A_1 e^{-(\gamma-q)t} + A_2 e^{-(\gamma+q)t} \qquad 3.4.1$$

where $q = (\gamma^2 - \omega_0^2)^{1/2}$

critical damping occurs when $q=0$ or $\gamma_c = \omega_0$.

So we have $\gamma = \frac{\omega_0}{2}$ and $\omega = 2\omega_0$ in a damped, driven oscillator

a)

$$\omega_r = \left[\omega_0^2 - 2\left(\frac{\omega_0}{2}\right)^2\right]^{1/2} = \frac{\omega_0}{\sqrt{2}}$$

b) $$Q = \frac{\omega_d}{2\gamma} \qquad 3.4.24$$

where $$\omega_d = \sqrt{\omega_0^2 - \gamma^2} \qquad 3.4.10$$

$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma} = \frac{\sqrt{\omega_0^2 - \frac{\omega_0^2}{4}}}{\omega_0} = 0.866$$

c) $$\tan\phi = \frac{2\gamma\omega}{(\omega_0^2 - \omega^2)} \qquad 3.6.20$$

$$\tan\phi = \frac{2\,\frac{\omega_0}{2}\,2\omega_0}{(\omega_0^2 - 4\omega_0^2)} = -\frac{2}{3}\ ; \quad \phi = -33.7^\circ$$

d) $$D(\omega = 2\omega_0) = \left[(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2\right]^{1/2}$$

$$= \left[(\omega_0^2 - 4\omega_0^2)^2 + 4\,\frac{\omega_0^2}{4}\,4\omega_0^2\right]^{1/2} = \sqrt{13}\ \omega_0^2$$

$$A = \frac{F_0/m}{D} = \frac{F_0}{\sqrt{13}\ m\omega_0^2}$$

3-14 $\gamma = \frac{1}{2}\gamma_c = \frac{\omega_0}{2}$; $\omega_1 = 2\omega_0$

[Note: is $2\omega_0 \approx \omega_0$?
I don't think so.]

a) $\omega_r = \sqrt{\omega_0^2 - 2\left(\frac{\omega_0}{2}\right)^2}$

$= \sqrt{\omega_0^2 - \frac{\omega_0^2}{2}}$

$= \frac{\omega_0}{\sqrt{2}}$

b) $Q = \frac{\omega_0}{2\gamma} = \frac{\omega_0}{\omega_0} = 1$

c) $\phi = \tan^{-1}\left(\frac{2\gamma\omega}{\omega_0^2 - \omega^2}\right)$

$= \tan^{-1} \frac{2\,\frac{\omega_0}{2}\,2\omega_0}{\omega_0^2 - 4\omega_0^2} =$

$= \tan^{-1} \frac{2}{-3}$

$\phi = -33.7^\circ$

d) $D(\omega_1 = 2\omega_0) = \left[\left(\omega_0^2 - \omega^2\right)^2 + 4\gamma^2\omega^2\right]^{1/2}$

$= \left[\left(\omega_0^2 - 4\omega_0^2\right)^2 + 4\,\frac{\omega_0^2}{4}\,4\omega_0^2\right]^{1/2}$

$= \sqrt{13}\;\omega_0^2$

$A = \frac{F_0/m}{D} = \frac{F_0}{\sqrt{13}\,m\,\omega_0^2} = \frac{0.277\,F_0}{m\,\omega_0^2}$

J-18 Driven Damped Harmonic Oscillator

2^{nd} law:

$$m\ddot{x} + c\dot{x} + kx = F_{ext}(t)$$

where $F_{ext} = F_0 e^{-\alpha t}\cos\omega t = F_0 \mathrm{Re}(e^{\beta t})$ with $\beta = -\alpha + i\omega$.

$$m\ddot{x} + c\dot{x} + hx = F_0 e^{\beta t}$$

Assume $x = Ae^{\beta t - i\phi}$, substitute into the diff. eq.:

$$\dot{x} = \beta A e^{\beta t - i\phi}$$
$$\ddot{x} = \beta^2 A e^{\beta t - i\phi}$$

$$m\beta^2 Ae^{\beta t - i\phi} + c\beta Ae^{\beta t - i\phi} + kAe^{\beta t - i\phi} = F_0 e^{\beta t}$$

$$m\beta^2 e^{-i\phi} + c\beta e^{-i\phi} + ke^{-i\phi} = \frac{F_0}{A}$$

$$m\beta^2 + c\beta + k = \frac{F_0}{A} e^{+i\phi}$$

put in $\beta = -\alpha + i\omega$, $\beta^2 = \alpha^2$.

$$m(\alpha^2 + 2i\alpha\omega - \omega^2) - c\alpha + ci\omega + k = \frac{F_0}{A} e^{+i\phi}$$

equate real + imaginary parts

$\cancel{m(\alpha^2 - \omega^2) - \ldots}$

3-18

real $m(\alpha^2 - \omega^2) - c\alpha + k = \frac{F_0}{A}\cos\phi$

imaginary $2m\alpha\omega + c\omega = \frac{F_0}{A}\sin\phi$

The unknowns are A & ϕ !

$$\tan\phi = \frac{2m\alpha\omega + c\omega}{m(\alpha^2-\omega^2) - c\alpha + k}$$

$$\left(\frac{F_0}{A}\right)^2(\sin^2\phi + \cos^2\phi) = (2m\alpha\omega + c\omega)^2 + m^2(\alpha^2-\omega^2) - c^2\alpha^2 + k^2$$

$$A^2 = \frac{F_0^2}{(2m\alpha\omega + c\omega)^2 + m^2(\alpha^2-\omega^2) - c^2\alpha^2 + k^2}$$

That's enough for me.