Problem 4.1. (Ideal gas engine with rectangular PV cycle.)

(a) The net work done by the gas during one cycle is

$$|W| = (P_2 - P_1)(V_2 - V_1) = (P_1)(2V_1) = 2P_1V_1,$$

while the heat absorbed (during steps A and B) is

$$Q_h = \frac{5}{2}V_1(P_2 - P_1) + \frac{7}{2}P_2(V_2 - V_1) = \frac{5}{2}V_1P_1 + 14P_1V_1 = \frac{33}{2}P_1V_1.$$

Therefore the efficiency is

$$e = \frac{|W|}{Q_h} = \frac{2P_1V_1}{\frac{33}{2}P_1V_1} = \frac{4}{33} = 12\%.$$

(b) The relative temperatures at various points around the cycle can be determined from the ideal gas law, $PV = NkT$. The lowest temperature occurs at the bottom-left corner when P and V are both smallest. As the pressure doubles during step A the temperature also doubles; then as the volume is tripled during step B so is the temperature. Thus the highest temperature, at the upper-right corner, is six times as great as the lowest temperature. For these extreme temperatures the maximum possible efficiency would be

$$e_{\text{max}} = 1 - \frac{T_c}{T_h} = 1 - \frac{T_c}{6T_c} = \frac{5}{6} = 83\%.$$

The rectangular cycle is extremely inefficient compared to a Carnot cycle.

Problem 4.3. (Waste heat from a power plant.)

(a) An efficiency of 40% means that the other 60% of the energy consumed ends up as waste heat. That's 1.5 times as much as the amount that ends up as work. More generally, by the definition of efficiency and the first law,

$$e = \frac{W}{Q_h} = \frac{W}{Q_c + W},$$

so the waste heat is

$$Q_c = W\left(\frac{1}{e} - 1\right) = 1.5\,W = 1.5\text{ GW}.$$

(b) In one second, the waste heat dumped to the river is 1.5×10^9 J, and this heat is spread among 10^5 kg of water, so each kilogram gets 15 kJ. With a heat capacity of 4186 J/°C, the water's temperature increases by $\Delta T = Q/C = 15000\text{ J}/4186\text{ J/°C} = 3.6\text{°C}$.

(c) The latent heat to evaporate water is 2260 J/g (at 100°C). At room temperature it's about 8% more, as mentioned in Problem 1.54 and Figure 5.11; so I'll take $L =$ 2400 J/g. The total amount of water that must evaporate each second is then

$$\frac{1.5 \times 10^9\text{ J}}{2400\text{ J/g}} = 6 \times 10^5\text{ g} = 600\text{ kg}.$$

That's only 0.6 m^3, or only 0.6% of the water in the river.

Problem 4.4. (Engine driven by the ocean's thermal gradient.)

(a) Converting the temperatures to the kelvin scale, we get a maximum possible efficiency of

$$e = 1 - \frac{T_c}{T_h} = 1 - \frac{277\text{ K}}{295\text{ K}} = 0.061,$$

or about 6%.

(b) A rigorous calculation of the absolute minimum amount of water that we must process is not easy. As the engine extracts heat from the warm water, the water's temperature decreases and therefore so does the efficiency of the engine. To make a rough estimate, however, let's suppose that we extract heat from the warm water until its temperature drops by 9°C (half the temperature difference between the warm and cool water), and similarly that we expel heat into the cool water until its temperature increases by 9°C. Then the average temperatures of the reservoirs are 290.5 K and 281.5 K, so the efficiency is only

$$e = 1 - \frac{281.5}{290.5} = 0.031.$$

The heat extracted from each kilogram of the warm water is 9×4186 J = 38 kJ, but at 3.1% efficiency, this heat produces only 1.2 kJ of work. We need 10^9 J of work each second, so the amount of water required is

$$\frac{10^9\text{ J}}{1200\text{ J/kg}} = 8.6 \times 10^5\text{ kg},$$

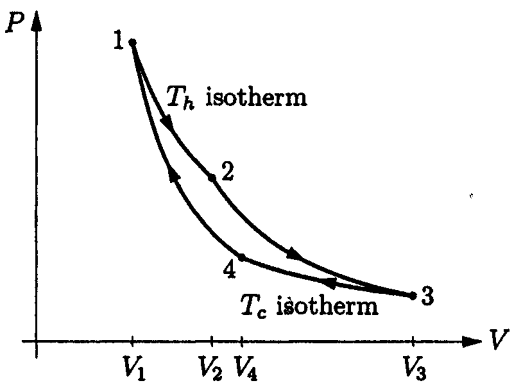
or about 900 cubic meters.

Problem 4.9. Suppose that the air conditioner must maintain a temperature of 20°C inside the building, while the outside temperature is 35°C. Then the maximum possible COP would be

$$\text{COP} = \frac{T_c}{T_h - T_c} = \frac{293\text{ K}}{15\text{ K}} = 19.5.$$

However, this theoretical maximum is unrealistically high, because real air conditioners are designed to work under more extreme temperatures, and to cool the air quickly rather than efficiently.

Problem 4.5. (Efficiency of an ideal gas Carnot engine.)



To compute Q_h and Q_c we need consider only the isothermal processes 1–2 and 3–4, since the other two steps are adiabatic. Furthermore, the heat input during an isothermal process is equal in magnitude to the work performed, since for an ideal gas $\Delta U \propto \Delta T = 0$. Therefore the heat input is

$$|Q_h| = |W_{12}| = \int_{V_1}^{V_2} P\,dV = NkT_h \ln \frac{V_2}{V_1},$$

and similarly,

$$|Q_c| = |W_{34}| = \int_{V_4}^{V_3} P\,dV = NkT_c \ln \frac{V_3}{V_4}.$$

The efficiency of the engine is

$$e = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{T_c}{T_h}\frac{\ln(V_3/V_4)}{\ln(V_2/V_1)},$$

which is equal to the Carnot efficiency provided that $V_3/V_4 = V_2/V_1$. To show that this is the case, note from equation 1.39 that for each of the adiabatic processes, $VT^{f/2}$ is constant

(where f is the number of degrees of freedom per molecule). For the adiabatic expansion 2–3, this implies

$$V_3 T_c^{f/2} = V_2 T_h^{f/2},$$

while for the adiabatic compression 4–1 we have

$$V_4 T_c^{f/2} = V_1 T_h^{f/2}.$$

Dividing these two equations, we obtain $V_3/V_4 = V_2/V_1$, as needed to cancel the logarithms in the preceding formula for the efficiency.

Problem 4.8. If you open the door of your refrigerator, the kitchen will initially cool down somewhat as the cool air from inside the fridge mixes with the warm air in the room. But then, as the refrigerator tries to suck heat out of its interior, it will dump *more* waste heat into your kitchen. So the long-term effect will actually be to increase the temperature of the kitchen, as the refrigerator tries in vain to cool the same space where it is dumping its waste heat.

Problem 4.10. As computed in the text, an ideal kitchen refrigerator could have a COP of about

$$\text{COP} = \frac{T_c}{T_h - T_c} = \frac{255\text{ K}}{298\text{ K} - 255\text{ K}} = 5.9.$$

That's cold!

Therefore, by the definition of COP, $Q_c = 5.9W$ or $W = Q_c/5.9$. In each second, this refrigerator must remove 300 J of heat from the inside, so the work required is $W = 300\text{ J}/5.9 = 50\text{ J}$. In other words, the power drawn from the wall could be as little as 50 W. (In practice the operation won't be ideal, of course.)

Problem 4.11. For the temperatures given, the maximum COP would be

$$\text{COP} = \frac{T_c}{T_h - T_c} = \frac{0.01\text{ K}}{1\text{ K} - 0.01\text{ K}} = 0.01.$$

In other words, for each joule of heat extracted from the very cold reservoir, we must supply at least 100 J (or 99, to be precise) of work.

Problem 4.14. The heat pump is physically the same as an ordinary refrigerator, so please refer to the energy-flow diagram in Figure 4.4.

(a) The COP should be defined as the benefit divided by the cost. In this case the benefit is the heat that enters the building, Q_h, while the cost is the electrical energy consumed, W. So benefit/cost $= Q_h/W$.

(b) The energy in is $Q_c + W$ and the energy out is Q_h, so

$$Q_h = Q_c + W$$

under cyclic operation. The COP is therefore

$$\text{COP} = \frac{Q_h}{Q_h - Q_c} = \frac{1}{1 - Q_c/Q_h},$$

which is *always* greater than 1.

(c) The entropy expelled during the cycle must be at least as great as the entropy absorbed, so

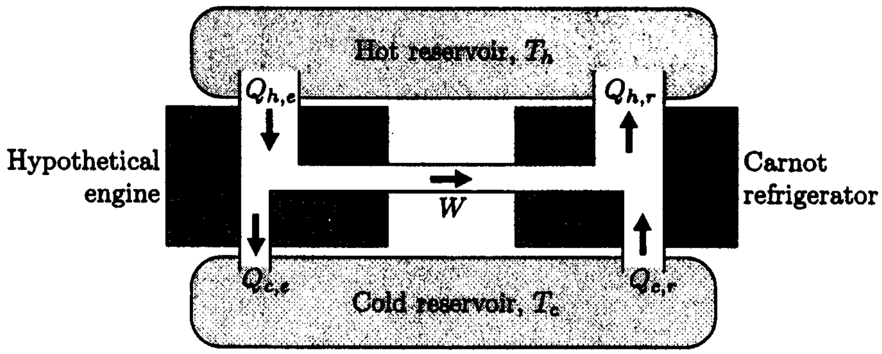
$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c} \quad \text{or} \quad \frac{T_c}{T_h} \geq \frac{Q_c}{Q_h}.$$

Because Q_c/Q_h must be *less* than or equal to T_c/T_h, the quantity $1 - Q_c/Q_h$ must be *greater* than or equal to $1 - T_c/T_h$, and therefore, by the result of part (b),

$$\text{COP} \leq \frac{1}{1 - T_c/T_h} = \frac{T_h}{T_h - T_c}.$$

(d) For an electric heater, all the electrical energy (W) is converted to heat (Q_h), so the COP is 1. An ideal heat pump, though, always has a COP greater than 1. For instance, if $T_h = 25°\text{C}$ and $T_c = 0°\text{C}$, then the COP can (in principle) be as high as $298/25 \approx 12$. In practice the COP is never this high, but as long as T_h and T_c aren't too different, a heat pump offers a big advantage in efficiency over an electric heater. On the other hand, a heat pump is more expensive to manufacture and maintain, since it a complicated device with many moving parts. Fortunately, a central air conditioning system can double as a heat pump in the winter. So if you're already planning to install central air, and your winters aren't *too* cold, get a heat pump.

Problem 4.16. Hook up the hypothetical engine to the Carnot refrigerator as shown below, so each uses the same reservoirs and the refrigerator uses all the work produced by the engine:



For the ideal Carnot refrigerator, the heat input and output are in the same ratio as the reservoir temperatures:

$$\frac{Q_{h,r}}{Q_{c,r}} = \frac{T_h}{T_c}.$$

For a given W, this equality and energy conservation ($Q_{h,r} - Q_{c,r} = W$) determine the values of $Q_{h,r}$ and $Q_{c,r}$. If the hypothetical engine were ideal, the same equalities would apply to it, so we would have $Q_{h,e} = Q_{h,r}$ and $Q_{c,e} = Q_{c,r}$. However, if the hypothetical engine is better than ideal, then it requires a smaller amount of heat input to produce the same amount of work, so $Q_{h,e} < Q_{h,r}$. Furthermore, energy conservation dictates that its waste heat output must be smaller by the same amount, so $Q_{c,e} < Q_{c,r}$. Thus, the net effect of the engine-refrigerator combination is to transfer heat (in an amount equal to $Q_{h,r} - Q_{h,e}$) from the cold reservoir to the hot reservoir, with no work input. It is a "perfect" refrigerator, too good to be true. We are therefore forced to conclude that no such hypothetical engine could possibly exist.

Problem 4.26. The net work done by the Rankine cycle is

$$W = Q_h - Q_c = (H_3 - H_2) - (H_4 - H_1) \approx H_3 - H_4,$$

where in the last step I've approximated $H_2 \approx H_1$ as in the text. For a kilogram of steam under the conditions assumed in the text, this is

$$W = 3444 \text{ kJ} - 1824 \text{ kJ} = 1620 \text{ kJ}.$$

To generate 10^9 joules (in one second), the number of kilograms of steam required would therefore be

$$\frac{10^9 \text{ J}}{1,620,000 \text{ J/kg}} = 617 \text{ kg}.$$

If we take into account the 30-kJ difference between H_2 and H_1 (as calculated in Problem 4.23), the work done per kilogram is reduced by 30 kJ and so the number of kilograms required is increased to $10^9/1,590,000 = 630$.

Problem 4.22. To compute the efficiency we need to know H_1, H_3, and H_4. At point 1 we have "saturated" (i.e, on the verge of boiling) liquid water at 20°C, so from Table 4.1 we see that the pressure must be 0.023 bar and the enthalpy (per kilogram) is 84 kJ. At point 3 we have "superheated" steam at 300°C and 10 bars, so from Table 4.2 we see that the enthalpy is 3051 kJ (per kilogram) and the entropy is 7.123 kJ/K. The expansion in the turbine is approximately adiabatic, so the entropy at point 4 should be the same as at point 3. At point 4, however, we have a mixture of saturated water ($S = 0.297$ kJ/K) and saturated steam ($S = 8.667$ kJ/K) at 20°C. If x is the fraction of water, then we require

$$7.123 = x(0.297) + (1-x)(8.667), \qquad \text{or} \qquad x = \frac{8.667 - 7.123}{8.667 - 0.297} = 0.184.$$

This same mixture has an enthalpy of

$$H_4 = (0.184)(84 \text{ kJ}) + (0.816)(2538 \text{ kJ}) = 2085 \text{ kJ}.$$

So, finally, the efficiency of our engine is

$$e \approx 1 - \frac{H_4 - H_1}{H_3 - H_1} = 1 - \frac{2085 - 84}{3051 - 84} = 0.33,$$

significantly less than that of the higher-temperature engine considered in the text.

Problem 4.23. Enthalpy is defined as $H = U + PV$. Therefore, under an infinitesimal change in conditions, the change in H is

$$dH = dU + P\,dV + V\,dP = T\,dS + V\,dP,$$

where in the second step I've used the thermodynamic identity (and set $dN = 0$ since we're interested in a fixed amount of fluid). Assuming that the compression of the water in step $1 \to 2$ is approximately adiabatic and quasistatic, the entropy doesn't change so $dH = V\,dP$. For our process the change in pressure isn't exactly infinitesimal, but the volume is still almost constant so we can write $\Delta H_{12} = V\,\Delta P_{12}$. For one kilogram of water taken from very low pressure up to 300 bars, this is

$$\Delta H_{12} = (10^{-3}\text{ m}^3)(300 \times 10^5\text{ N/m}^2) = 30\text{ kJ},$$

so the enthalpy at point 2 is approximately 114 kJ, rather than 84 kJ as approximated in equation 4.13. With this improved estimate of H_2, the efficiency would be

$$e = 1 - \frac{1824 - 84}{3444 - 114} = 0.477,$$

rather than 0.482 as you would get if you set $H_2 = H_1 = 84$ kJ. But this difference is only half a percent, and two significant figures is probably all that would be justified in any case.

Problem 4.30. (Household refrigerator.)

(a) The entropy at point 1, for a kilogram of fluid, is 940 J/K (from Table 4.3). Looking at the 10-bar row of Table 4.4, note that at 50°C the entropy would be 943 J/K, only slightly higher. Assuming that S is a linear function of temperature between 40° and 50°, it would increase by 3.6 J/K per degree, or 3 J/K in 0.83 degrees. Therefore the temperature at point 2 must be 0.83 degrees less than 50°, or just over 49°C.

(b) From Table 4.3, $H_1 = 231$ kJ. To find H_2, repeat the same interpolation in Table 4.4: Between 40 and 50 degrees the enthalpy increases by 2.1 kJ per degree, so 0.83 degrees below 50 the enthalpy should be less than 280 kJ by (0.83)(2.1 kJ) = 1.75 kJ, that is, $H_2 = 278.25$ kJ ≈ 280 kJ. At point 3 the fluid is saturated liquid at 10 bars, so $H_3 = 105$ kJ from Table 4.3. And since the throttling process leaves the enthalpy unchanged, $H_4 = 105$ kJ as well. Plugging these results into equation 4.20, we obtain for the coefficient of performance

$$\mathrm{COP} = \frac{H_1 - H_3}{H_2 - H_1} = \frac{231 - 105}{280 - 231} = 2.67.$$

To compare to a Carnot refrigerator operating between the same reservoir temperatures, note that the high-temperature reservoir can be no hotter than $T_3 = 39.4°\mathrm{C} = 312.6$ K, while the low-temperature reservoir can be no colder than $T_4 = T_1 = -26.4°\mathrm{C} = 246.8$ K. For this temperature range the Carnot COP would be

$$\frac{T_c}{T_h - T_c} = \frac{246.8}{312.6 - 246.8} = 3.75,$$

only moderately better than the actual efficiency of the cycle. To increase the efficiency we could reduce T_h to a value only slightly higher than the temperature of the kitchen (32°C?), and/or increase T_c to a value only slightly lower than the temperature of the freezer (−15°C?). However, for reasonably rapid heat transfer between the refrigerant and either reservoir, we should probably have a temperature difference of at least 10°C; therefore the temperatures assumed in this example are probably about right.

(c) Setting the initial enthalpy equal the enthalpy of the unknown final mixture, we have (with all numbers in kJ)

$$105 = 16x + 231(1 - x) \qquad \text{or} \qquad x = \frac{231 - 105}{231 - 16} = 0.59,$$

where x is the fraction that remains liquid. So 41% of the fluid (by mass) vaporizes during the throttling.