```
vcar=v0car+acar*t;                     Calculate the car's velocity.
speed_trainRcar=sqrt(vcar.^2+v0train^2);
                         Calculate the speed of the train relative to the car.
table=[t' y' x' d' vcar' speed_trainRcar']
                                      Create a table (see note below).
```

Note: In the commands above, `table` is the name of the variable that is a matrix containing the data to be displayed.

When the script file is executed, the following is displayed in the Command Window:

```
table =
         0 -400.0000 -200.0000  447.2136   41.0667   89.2139
    1.0000 -320.8000 -156.9333  357.1284   45.0667   91.1243
    2.0000 -241.6000 -109.8667  265.4077   49.0667   93.1675
    3.0000 -162.4000  -58.8000  172.7171   53.0667   95.3347
    4.0000  -83.2000   -3.7333   83.2837   57.0667   97.6178
    5.0000   -4.0000   55.3333   55.4777   61.0667  100.0089
    6.0000   75.2000  118.4000  140.2626   65.0667  102.5003
    7.0000  154.4000  185.4667  241.3239   69.0667  105.0849
    8.0000  233.6000  256.5333  346.9558   73.0667  107.7561
    9.0000  312.8000  331.6000  455.8535   77.0667  110.5075
   10.0000  392.0000  410.6667  567.7245   81.0667  113.3333
```

Time (s)	Train position (ft)	Car position (ft)	Car-train distance (ft)	Car speed (ft/s)	Train speed relative to the car (ft/s)

In this problem the results (numbers) are displayed by MATLAB without any text. Instructions on how to add text to output generated by MATLAB are presented in Chapter 4.

3.9 Problems

Note: Additional problems for practicing mathematical operations with arrays are provided at the end of Chapter 4.

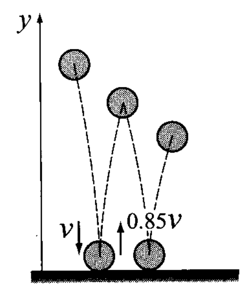
1. For the function $y = x^3 - 2x^2 + x$, calculate the value of y for the following values of x using element-by-element operations: $-2, -1, 0, 1, 2, 3, 4$.

2. For the function $y = \dfrac{x^2 - 2}{x + 4}$, calculate the value of y for the following values of x using element-by-element operations: $-3, -2, -1, 0, 1, 2, 3$.

3. For the function $y = \frac{(x-3)(x^2+3)}{x^2}$, calculate the value of y for the following values of x using element-by-element operations: $1, 2, 3, 4, 5, 6, 7$.

4. For the function $y = \frac{20t^{2/3}}{t+1} - \frac{(t+1)^2}{e^{(0.3t+5)}} + \frac{2}{t+1}$, calculate the value of y for the following values of t using element-by-element operations: $0, 1, 2, 3, 4, 5, 6, 7, 8$.

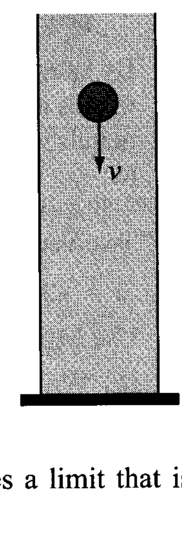
5. A ball that is dropped on the floor bounces back up many times, reaching a lower height after each bounce. When the ball impacts the floor its rebound velocity is 0.85 times the impact velocity. The velocity v with which a ball hits the floor after being dropped from a height h is given by $v = \sqrt{2gh}$, where $g = 9.81$ m/s^2. The time between successive bounces is given by $t = v/g$, where v is the upward velocity after the last impact. Consider a ball that is dropped from a height of 2 m. Determine the times at which the ball hits the floor for the first eight bounces. Set $t = 0$ when the ball hits the floor for the first time. (Calculate the velocity of the ball when it hits the floor for the first time. Derive a formula for the time of the following hits as a function of the bounce number. Then create a vector $n = 1, 2, \ldots, 8$ and use the formula (use element-by element operations) to calculate a vector with the values of t for each n.) Display the results in a two-column table where the values of n and t are displayed in the first and second columns, respectively.



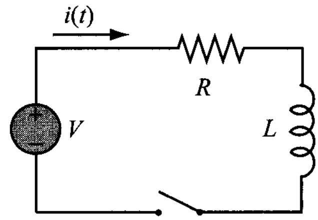
6. An aluminum sphere ($r = 0.2$ cm) is dropped in a glass cylinder filled with glycerin. The velocity of the sphere as a function of time $v(t)$ can be modeled by the equation

$$v(t) = \sqrt{\frac{V(\rho_{al}-\rho_{gl})g}{k}}\tanh\left(\frac{\sqrt{V(\rho_{al}-\rho_{gl})gk}}{V\rho_{al}}t\right)$$

where V is the volume of the sphere, $g = 9.81$ m/s^2 is the gravitational acceleration, $k = 0.0018$ is a constant, and $\rho_{al} = 2700$ kg/m^3 and $\rho_{gl} = 1260$ kg/m^3 are the density of aluminum and glycerin, respectively. Determine the velocity of the sphere for $t = 0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3,$ and 0.35 s. Note that initially the velocity increases rapidly, but then, due to the resistance of the glycerin, the velocity increases more gradually. Eventually the velocity approaches a limit that is called the terminal velocity.



7. The current i (in amps) t seconds after closing the switch in the circuit shown is given by:



$$i(t) = \frac{V}{R}(1 - e^{-(R/L)\,t})$$

Consider the case where $V = 120$ volts, $R = 120$ ohms and $L = 0.1$ henry.

(*a*) Find the time t_m required for the current to reach 1% of its initial value, then use `linspace` to create a vector t having 10 elements with the first element 0 and maximum value t_m.

(*b*) Calculate the current i for each value of t from part (*a*).

8. The length $|\mathbf{u}|$ (magnitude) of a vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is given by $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$. Given the vector $\mathbf{u} = 23.5\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$, determine its length two ways:

(*a*) Define the vector in MATLAB, and then write a mathematical expression that uses the components of the vector.

(*b*) Define the vector in MATLAB, then use element-by element operations to create a new vector with elements that are the squares of the elements of the original vector. Then use MATLAB built-in functions `sum` and `sqrt` to calculate the length. All of these steps can be written in one command.

9. The unit vector $\mathbf{u}_n$ in the direction of the vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is given by $\mathbf{u}_n = \dfrac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{\sqrt{x^2 + y^2 + z^2}}$. Determine the unit vector of the vector $\mathbf{u} = -8\mathbf{i} - 14\mathbf{j} + 25\mathbf{k}$ by writing one MATLAB command.

10. The following two vectors are defined in MATLAB:

$$v = [3, -2, 4] \qquad u = [5, 3, -1]$$

By hand (pencil and paper) write what will be displayed if the following commands are executed by MATLAB. Check your answers by executing the commands with MATLAB.

(*a*) `v.*u` (*b*) `v*u'` (*c*) `v'*u`

11. Two vectors are given:

$$\mathbf{u} = -3\mathbf{i} + 8\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \mathbf{v} = 6.5\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}$$

Use MATLAB to calculate the dot product $\mathbf{u} \cdot \mathbf{v}$ of the vectors in three ways:

(*a*) Write an expression using element-by-element calculation and the MATLAB built-in function `sum`.

(*b*) Define $\mathbf{u}$ as a row vector and $\mathbf{v}$ as a column vector, and then use matrix multiplication.

(*c*) Use the MATLAB built-in function `dot`.

12. Define the vector $v = [2\ \ 4\ \ 6\ \ 8\ \ 10]$. Then use the vector in a mathematical expression to create the following vectors:

(a) $a = \left[\frac{1}{2}\ \frac{1}{4}\ \frac{1}{6}\ \frac{1}{8}\ \frac{1}{10}\right]$ (b) $b = \left[\frac{1}{2^2}\ \frac{1}{4^2}\ \frac{1}{6^2}\ \frac{1}{8^2}\ \frac{1}{10^2}\right]$

(c) $c = [1\ \ 2\ \ 3\ \ 4\ \ 5]$ (d) $d = [1\ \ 1\ \ 1\ \ 1\ \ 1]$

13. Define the vector $v = [5\ \ 4\ \ 3\ \ 2\ \ 1]$. Then use the vector in a mathematical expression to create the following vectors:

(a) $a = [5^2\ \ 4^2\ \ 3^2\ \ 2^2\ \ 1^2]$ (b) $b = [5^5\ \ 4^4\ \ 3^3\ \ 2^2\ \ 1^1]$

(c) $c = [25\ \ 20\ \ 15\ \ 10\ \ 5]$ (d) $d = [4\ \ 3\ \ 2\ \ 1\ \ 0]$

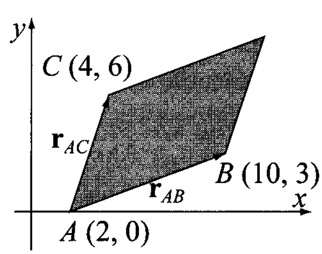
14. Define x and y as the vectors $x = [1, 3, 5, 7, 9]$ and $y = [2, 5, 8, 11, 14$. Then use them in the following expressions to calculate z using element-by-element calculations.

(a) $z = \frac{xy^2}{x+y}$ (b) $z = x(x^2-y)-(x-y)^2$

15. Define p and w as scalars, $p = 2.3$ and define $w = 5.67$, and, t, x, and y as the vectors $t = [1, 2, 3, 4, 5]$, $x = [2.8, 2.5, 2.2, 1.9, 1.6]$, and $y = [4, 7, 10, 13, 17]$. Then use these variables to calculate the following expressions using element-by-element calculations for the vectors.

(a) $T = \frac{p(x+y)^2}{y}w$ (b) $S = \frac{p(x+y)^2}{yw} + \frac{wxt}{py}$

16. The area of the parallelogram shown can be calculated by $|\mathbf{r}_{AB} \times \mathbf{r}_{AC}|$. Use the following steps in a script file to calculate the area:
Define the position of points A, B, and C as vectors $A = [2, 0]$, $B = [10, 3]$, and $C = [4, 6]$.
Determine the vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$ from the points.
Determine the area by using MATLAB's built-in functions `cross`, `sum`, and `sqrt`.



17. Define the vectors:

$$\mathbf{u} = -2\mathbf{i}+6\mathbf{j}+5\mathbf{k},\ \mathbf{v} = 5\mathbf{i}-1\mathbf{j}+3\mathbf{k},\ \text{and}\ \mathbf{w} = 4\mathbf{i}+7\mathbf{j}-2\mathbf{k}$$

Use the vectors to verify the identity:

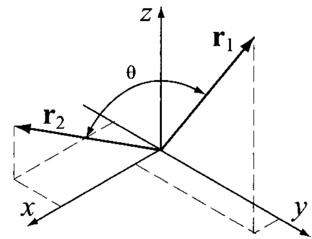
$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\mathbf{u}\cdot\mathbf{w}) - \mathbf{w}(\mathbf{u}\cdot\mathbf{v})$$

Using MATLAB's built-in functions `cross` and `abs`, calculate the value of the left and right sides of the identity.

18. The dot product can be used for determining the angle between two vectors:

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1||\mathbf{r}_2|}\right)$$

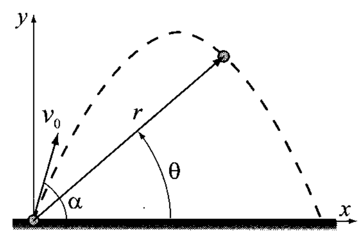
Use MATLAB's built-in functions `cosd`, `sqrt`, and `dot` to find the angle (in degrees) between $\mathbf{r}_1 = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{r}_2 = 1\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$.
Recall that $|\mathbf{r}| = \sqrt{\mathbf{r} \cdot \mathbf{r}}$.



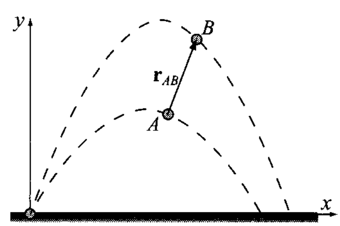
19. The position as a function of time $(x(t), y(t))$ of a projectile fired with a speed of v_0 at an angle α is given by

$$x(t) = v_0 \cos\alpha \cdot t \qquad y(t) = v_0 \sin\alpha \cdot t - \frac{1}{2}gt^2$$

where $g = 9.81$ m/s^2. The polar coordinates of the projectile at time t are $(r(t), \theta(t))$, where $r(t) = \sqrt{x(t)^2 + y(t)^2}$ and $\tan\theta = \frac{y(t)}{x(t)}$. Consider the case where $v_0 = 162$ m/s and $\theta = 70°$. Determine $r(t)$ and $\theta(t)$ for $t = 1, 6, 11, \ldots, 31$ s.



20. Two projectiles, A and B, are shot at the same instant from the same spot. Projectile A is shot at a speed of 560 m/s at an angle of 43° and projectile B is shot at a speed of 680 m/s at an angle of 50°. Determine which projectile will hit the ground first. Then take the flying time t_f of that projectile and divide it into ten increments by creating a vector t with 11 equally spaced elements (the first element is 0, the last is t_f). At each time t calculate the position vector $\mathbf{r}_{AB}$ between the two projectiles. Display the results in a three-column matrix where the first column is t and the second and third columns are the corresponding x and y components of $\mathbf{r}_{AB}$.



21. Show that $\lim_{x \to 0} \frac{\sin x}{x} = 1$.
Do this by first creating a vector x that has the elements 1.5, 1.0, 0.5, 0.1, 0.01, 0.001, and 0.00001. Then, create a new vector y in which each element is determined from the elements of x by $\frac{\sin x}{x}$. Compare the elements of y with the value 1 (use format long to display the numbers).

22. Show that $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$.
Do this by first creating a vector x that has the elements: 5, 3, 2, 1.5, 1.1, 1.001, and 1.00001. Then, create a new vector y in which each element is determined from the elements of x by $\frac{x^2 - 1}{x - 1}$. Compare the elements of y with the value 2 (use format long to display the numbers).

23. Use MATLAB to show that the sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots$ converges to 1. Do it by computing the sum for:
(*a*) $n = 10$ (*b*) $n = 20$
(*c*) $n = 30$ (*c*) $n = 40$
For each part create a vector `n` in which the first element is 1, the increment is 1, and the last term is 10, 20, 30, or 40. Then use element-by-element calculations to create a vector in which the elements are $\frac{1}{2^n}$. Finally, use the MATLAB built-in function `sum` to add the terms of the series. Compare the values obtained in parts (*a*), (*b*), (*c*), and (*d*) with the value of 1. (Don't forget to type semicolons at the end of commands that otherwise will display large vectors.)

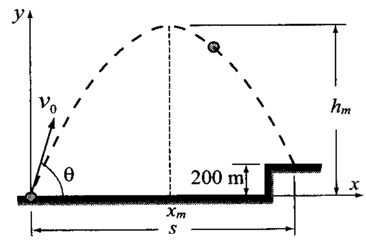
24. Use MATLAB to show that the sum of the infinite series $\sqrt{12}\sum_{n=0}^{\infty} \frac{(-3)^{-n}}{2n+1}$ is equal to π. Do this by computing the sum for:
(*a*) $n = 10$ (*b*) $n = 20$ (*c*) $n = 50$
For each part create a vector `n` in which the first element is 0, the increment is 1 and the last term is 10, 50, or 100. Then, use element-by-element calculation to create a vector in which the elements are $\frac{(-3)^{-n}}{2n+1}$. Finally, use the function `sum` to add the terms of the series and multiply the result by $\sqrt{12}$. Compare the values obtained in parts (*a*), (*b*), and (*c*) to the value of π in MATLAB.

25. Fisheries commonly estimate the growth of a fish population using the von Bertalanffy growth law:

$$L = L_{max}(1 - e^{-K(t+\tau)})$$

where L_{max} is the maximum length, K is a rate constant, and τ is a time constant. These constants vary with the species of fish. Assuming L_{max} = 58 cm, K = 0.45 years^{-1}, and τ = 0.65 years, calculate the length of a fish at 0, 1, 2, 3, 4, and 5 years of age.

26. The path of a projectile fired with an initial speed v_0 at an angle θ is described by the equation



$$y = x\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2$$

where $g = 9.81\,\text{m/s}^2$. Consider the case where $\theta = 75°$ and $v_0 = 110\,\text{m/s}$. Write a MATLAB script that does the following: calculates the distance s traveled by the projectile, creates a vector x with 100 elements such that the first element is 0 and the last is s, calculates the value of y for each value of x, finds the maximum height h_m that the projectile reaches (use MATLAB built-in function `max`) and the distance x_{hm} where the maximum height is reached. When the script is executed only the values of h_m and x_{hm} are displayed.

27. Create the following three matrices:

$$A = \begin{bmatrix} 2 & 4 & -1 \\ 3 & 1 & -5 \\ 0 & 1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 5 & 0 \\ -3 & 2 & 7 \\ -1 & 6 & 9 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 3 & 5 \\ 2 & 1 & 0 \\ 4 & 6 & -3 \end{bmatrix}$$

(*a*) Calculate $A + B$ and $B + A$ to show that addition of matrices is commutative.

(*b*) Calculate $A + (B + C)$ and $(A + B) + C$ to show that addition of matrices is associative.

(*c*) Calculate $5(A + C)$ and $5A + 5C$ to show that, when matrices are multiplied by a scalar, the multiplication is distributive.

(*d*) Calculate $A*(B + C)$ and $A*B + A*C$ to show that matrix multiplication is distributive.

28. Use the matrices A, B, and C from the previous problem to answer the following:

(*a*) Does $A*B = B*A$?

(*b*) Does $A*(B*C) = (A*B)*C$?

(*c*) Does $(A*B)^t = B^t*A^t$? (^{t} means transpose)

(*d*) Does $(A + B)^t = A^t + B^t$?

29. Create a 4×4 matrix having random integer values between 1 and 10. Call the matrix A and using MATLAB perform the following operations. For each part explain the operation.

(*a*) $A * A$

(*b*) $A .*A$

(*c*) $A \backslash A$

(*d*) $A .\backslash A$

(*e*) det(A)

(*e*) inv(A)

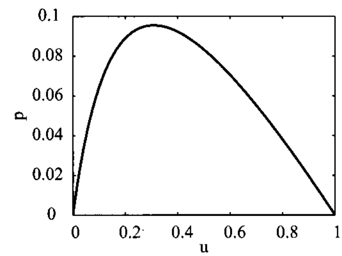
30. The mechanical power output P in a contracting muscle is given by

$$P = Tv = \frac{kvT_0\left(1 - \frac{v}{v_{max}}\right)}{k + \frac{v}{v_{max}}}$$

where T is the muscle tension, v is the shortening velocity (max of v_{max}), T_0 is the isometric tension (i.e., tension at zero velocity), and k is a non-dimensional constant that ranges between 0.15 and 0.25 for most muscles. The equation can be written in non-dimensional form:

$$p = \frac{ku(1-u)}{k+u}$$

where $p = (Tv)/(T_0 v_{max})$, and $u = v/v_{max}$. A figure with $k = 0.25$ is shown here.



(*a*) Create a vector u ranging from 0 to 1 with increments of 0.05.

(*b*) Using $k = 0.25$, calculate the value of p for each value of u.

(*c*) Using MATLAB built-in function `max`, find the maximum value of p.

(*d*) Repeat the first three steps with increments of 0.01 and calculate the percent relative error, defined by $E = \left|\frac{p_{max_{0.01}} - p_{max_{0.05}}}{p_{max_{0.05}}}\right| \times 100$.

31. Solve the following system of three linear equations:

$$\begin{aligned} 3x - 2y + 5z &= 7.5 \\ -4.5 + 2y + 3z &= 5.5 \\ 5x + y - 2.5z &= 4.5 \end{aligned}$$

32. Solve the following system of five linear equations:

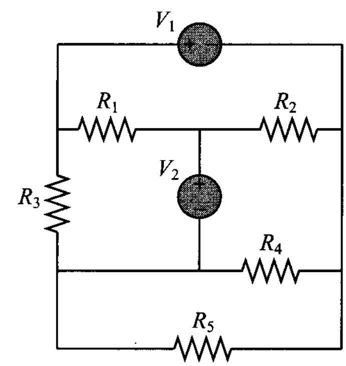
$$\begin{aligned} 3u + 1.5v + w + 0.5x + 4y &= -11.75 \\ -2u + v + 4w - 3.5x + 2y &= 19 \\ 6u - 3v + 2w + 2.5x + y &= -23 \\ u + 4v - 3w + 0.5x - 2y &= -1.5 \\ 3u + 2v - w + 1.5x - 3y &= -3.5 \end{aligned}$$

33. A juice company manufactures one-gallon bottles of three types of juice blends using orange, pineapple, and mango juice. The blends have the following compositions:
1 gallon orange blend: 3 quarts of orange juice, 0.75 quart of pineapple juice, 0.25 quart of mango juice.
1 gallon pineapple blend: 1 quart of orange juice, 2.5 quarts of pineapple juice, 0.5 quart of mango juice.

1 gallon mango blend: 0.5 quart of orange juice, 0.5 quart of pineapple juice, 3 quarts of mango juice.
How many gallons of each blend can be manufactured if 7,600 gallons of orange juice, 4,900 gallons of pineapple juice, and 3,500 gallons mango juice are available? Write a system of linear equations and solve.

34. The electrical circuit shown consists of resistors and voltage sources. Determine the current in each resistor, using the mesh current method based on Kirchhoff's voltage law (see Sample Problem 3-4).

$V_1 = 12\text{ V}$, $V_2 = 24\text{ V}$
$R_1 = 20\,\Omega$, $R_2 = 12\,\Omega$, $R_3 = 8\,\Omega$
$R_4 = 6\,\Omega$, $R_5 = 10\,\Omega$



35. The electrical circuit shown consists of resistors and voltage sources. Determine the current in each resistor, using the mesh current method based on Kirchhoff's voltage law (see Sample Problem 3-4).

$V_1 = 12\text{ V}$, $V_2 = 24\text{ V}$
$R_1 = 20\,\Omega$, $R_2 = 12\,\Omega$, $R_3 = 8\,\Omega$
$R_4 = 6\,\Omega$, $R_5 = 10\,\Omega$