

15.0000	3.7113	0.9183
10.0000	2.4742	0.6122

The current in the circuit is 0.247423 Amps.

The total power dissipated in the circuit is 5.938144 Watts.

4.7 PROBLEMS

Solve the following problems by first writing a program in a script file and then executing the program.

- The wind chill temperature, T_{wc} , is the air temperature felt on exposed skin due to wind. In U.S. customary units it is calculated by

$$T_{wc} = 35.74 + 0.6215T - 35.75v^{0.16} + 0.4275T v^{0.16}$$

where T is the temperature in degrees F and v is the wind speed in mi/h. Write a MATLAB program in a script file that calculates T_{wc} . For input the program asks the user to enter values for T and v . For output the program displays the message: "The wind chill temperature is: XX," where XX is the value of the wind chill temperature rounded to the nearest integer. Execute the program entering $T = 30^\circ\text{F}$ and $v = 42$ mi/h.

- The monthly payment M of a loan amount P for y years and with interest rate r can be calculated by the formula:

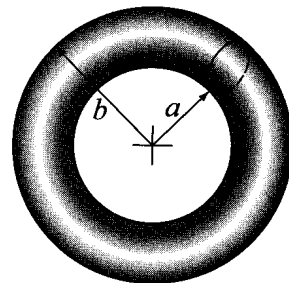
$$M = \frac{P(r/12)}{1 - (1 + r/12)^{-12y}}$$

Calculate the monthly payment and the total payment for a \$100,000 loan for 10, 11, 12, ..., 29, 30 years with an interest rate of 4.85%. Display the results in a three-column table where the first column is the number of years, the second is the monthly payment, and the third is the total payment.

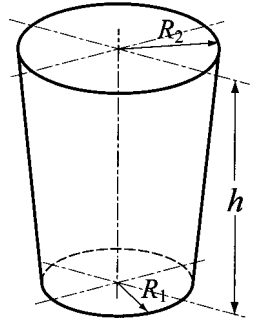
- A torus-shaped water tube is designed to have a volume of 8,000 in.³. The volume of the tube, V , and its surface area, S , are given by:

$$V = \frac{1}{4}\pi^2(a+b)(b-a)^2 \text{ and } S = \pi^2(b^2 - a^2)$$

If $a = Kb$, determine S and a and b for $K = 0.2, 0.3, 0.4, 0.6,$ and 0.7 . Display the results in a table.



4. An ice cream container shaped as a frustum of a cone with $R_2 = 1.2R_1$ is designed to have a volume of 1,000 cm^3 . Determine R_1 , R_2 , and the surface area, S , of the paper for containers with heights h of 8, 10, 12, 14, and 16 cm. Display the results in a table.



The volume of the container, V , and the surface area of the paper are given by:

$$V = \frac{1}{3}\pi h(R_1^2 + R_2^2 + R_1R_2)$$

$$S = \pi(R_1 + R_2)\sqrt{(R_2 - R_1)^2 + h^2} + \pi(R_1^2 + R_2^2)$$

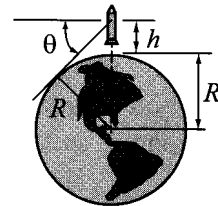
5. Write a MATLAB program in a script file that calculate the average, standard deviation, and median of a list of grades as well as the number of grades on the list. The program asks the user (`input` command) to enter the grades as elements of a vector. The program then calculates the required quantities using MATLAB's built-in functions `length`, `mean`, `std`, and `median`. The results are displayed in the Command Window in the following format:
 "There are XX grades." where XX is the numerical value.
 "The average grade is XX." where XX is the numerical value.
 "The standard deviation is XX." where XX is the numerical value.
 "The median deviation is XX." where XX is the numerical value.
 Execute the program and enter the following grades: 81, 65, 61, 78, 94, 80, 65, 76, 77, 95, 82, 49, and 75.

6. The growth of some bacteria populations can be described by

$$N = N_0e^{kt}$$

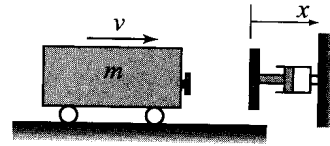
where N is the number of individuals at time t , N_0 is the number at time $t = 0$, and k is a constant. Assuming the number of bacteria doubles every hour, determine the number of bacteria every hour for 24 hours starting from an initial single bacterium.

7. A rocket flying straight up measures the angle θ with the horizon at different heights h . Write a MATLAB program in a script file that calculates the radius of the earth R (assuming the earth is a perfect sphere) at each data point and then determines the average of all the values.



h (km)	4	8	12	16	20	24	28	32	36	40
θ (deg)	2.0	2.9	3.5	4.1	4.5	5.0	5.4	5.7	6.1	6.4

8. A railroad bumper is designed to slow down a rapidly moving railroad car. After a 20,000 kg railroad car traveling at 20 m/s engages the bumper, its displacement x (in meters) and velocity v (in m/s) as a function of time t (in seconds) is given by:



$$x(t) = 4.219(e^{-1.58t} - e^{-6.32t}) \quad \text{and} \quad v(t) = 26.67e^{-6.32t} - 6.67e^{-1.58t}$$

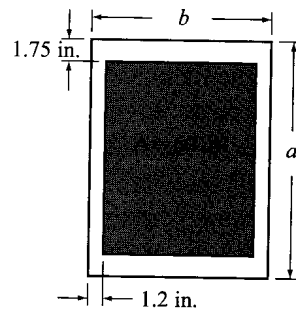
Determine x and v for every two hundredth of a second for the first half second after impact. Display the results in a three-column table in which the first column is time (s), the second is displacement (m), and the third is velocity (m/s).

9. Decay of radioactive materials can be modeled by the equation $A = A_0 e^{kt}$, where A is the amount at time t , A_0 is the amount at $t = 0$, and k is the decay constant ($k \leq 0$). Iodine-132 is a radioisotope that is used in thyroid function tests. Its half-life time is 13.3 hours. Calculate the relative amount of Iodine-132 (A/A_0) in a patient's body 48 hours after receiving a dose. After determining the value of k , define a vector $t = 0, 4, 8, \dots, 48$ and calculate the corresponding values of A/A_0 .
10. The value, B , of a savings account of an amount A that is deposited for n years with a yearly interest rate of r is given by:

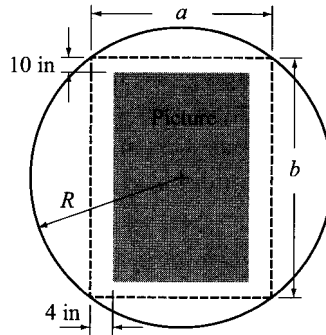
$$B = A \left(1 + \frac{r}{100} \right)^n$$

Write a MATLAB program in a script file that calculates the balance B after 10 years for an initial deposit of \$10,000 for yearly interest rates ranging from 2% to 6% with increments of 0.5%. Display the results in a table. The table should have two columns where the first column displays the interest rate and the second displays the corresponding value of B .

11. A rectangular printed page with sides of lengths a and b is designed to have a printed area of 60 in.² and margins of 1.75 in. at the top and bottom and 1.2 in. at both sides. Write a MATLAB program that determine the dimensions of a and b such that the overall area of the page will be as small as possible. In the program define a vector a with values ranging from 5 to 20 with increments of 0.05. Use this vector for calculating the corresponding values of b and the overall area of the page. Then use MATLAB's built-in function `min` to find the dimensions of the smallest page.



12. A round billboard with radius $R = 55$ in. is designed to have a rectangular picture placed inside a rectangle with sides a and b . The margins between the rectangle and the picture are 10 in. at the top and bottom and 4 in. at each side. Write a MATLAB program that determines the dimensions a and b such that the overall area of the picture will be as large as possible. In the program define a vector a with values ranging from 5 to 100 with increments of 0.25. Use this vector for calculating the corresponding values of b and the overall area of the picture. Then use MATLAB's built-in function `max` to find the dimensions of the largest rectangle.

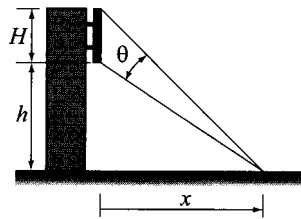


13. The balance of a loan, B , after n monthly payments is given by

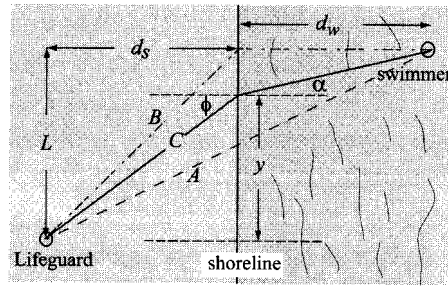
$$B = A \left(1 - \frac{r}{1200}\right)^n - \frac{P}{r/1200} \left[\left(1 + \frac{r}{1200}\right)^n - 1\right]$$

where A is the loan amount, P is the amount of a monthly payment, and r is the yearly interest rate entered in % (e.g., 7.5% entered as 7.5). Consider a 5-year, \$20,000 car loan with 6.5% yearly interest that has a monthly payment of \$391.32. Calculate the balance of the loan after every 6 months (i.e., at $n = 6, 12, 18, 24, \dots, 54, 60$). Each time calculate the percent of the loan that is already paid. Display the results in a three-column table, where the first column displays the month, and the second and third columns display the corresponding value of B and percentage of the loan that is already paid, respectively.

14. A large TV screen of height $H = 50$ ft is placed on the side wall of a tall building. The height from the street to the bottom of the screen is $h = 130$ ft. The best view of the screen is when θ is maximum. Write a MATLAB program that determines the distance x at which θ is at maximum. Define a vector x with elements ranging from 30 to 300 with spacing of 0.5. Use this vector to calculate the corresponding values of θ . Then use MATLAB's built-in function `min` to find the value of x that corresponds to the largest value of θ .



15. A student has a summer job as a lifeguard at the beach. After spotting a swimmer in trouble, he tries to deduce the path by which he can reach the swimmer in the shortest time. The path of shortest distance (path *A*) is obviously not the best since it maximizes the time spent swimming (he can run faster than he can swim).

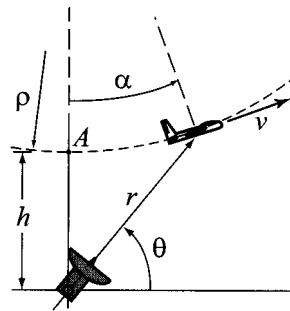


Path *B* minimizes the time spent swimming but is probably not the best since it is the longest (reasonable) path. Clearly the optimal path is somewhere in between paths *A* and *B*.

Consider an intermediate path *C* and determine the time required to reach the swimmer in terms of the running speed $v_{run} = 3 \text{ m/s}$ the swimming speed $v_{swim} = 1 \text{ m/s}$; the distances $L = 48 \text{ m}$, $d_s = 30 \text{ m}$, and $d_w = 42 \text{ m}$; and the lateral distance y at which the lifeguard enters the water. Create a vector y that ranges between path *A* and path *B* ($y = 20, 21, 22, \dots, 48 \text{ m}$) and compute a time t for each y . Use MATLAB built-in function `min` to find the minimum time t_{min} and the entry point y for which it occurs. Determine the angles that correspond to the calculated value of y and investigate whether your result satisfies Snell's law of refraction:

$$\frac{\sin \phi}{\sin \alpha} = \frac{v_{run}}{v_{swim}}$$

16. The airplane shown is flying at a constant speed of $v = 50 \text{ m/s}$ in a circular path of radius $\rho = 2000 \text{ m}$ and is being tracked by a radar station positioned a distance $h = 500 \text{ m}$ below the bottom of the plane path (point *A*). The airplane is at point *A* at $t = 0$, and the angle α as a function of time is given (in radians) by $\alpha = \frac{v}{\rho}t$. Write a MATLAB program



that calculates θ and r as functions of time. The program should first determine the time at which $\alpha = 90^\circ$. Then construct a vector t having 15 elements over the interval $0 \leq t \leq t_{90^\circ}$, and calculate θ and r at each time. The program should print the values of ρ , h , and v , followed by a 15×3 table where the first column is t , the second is the angle θ in degrees, and the third is the corresponding value of r .

17. Early explorers often estimated altitude by measuring the temperature of boiling water. Use the following two equations to make a table that modern-day hikers could use for the same purpose.

$$p = 29.921(1 - 6.8753 \times 10^{-6}h), \quad T_b = 49.161 \ln p + 44.932$$

where p is atmospheric pressure in inches of mercury, T_b is boiling temperature in $^{\circ}\text{F}$, and h is altitude in feet. The table should have two columns, the first altitude and the second boiling temperature. The altitude should range between -500 ft and $10,000$ ft at increments of 500 ft.

18. The variation of vapor pressure p (in units of mm Hg) of benzene with temperature in the range of $0 \leq T \leq 42^{\circ}\text{C}$ can be modeled with the equation (Handbook of Chemistry and Physics, CRC Press)

$$\log_{10} p = b - \frac{0.05223a}{T}$$

where $a = 34172$ and $b = 7.9622$ are material constants and T is absolute temperature (K). Write a program in a script file that calculates the pressure for various temperatures. The program should create a vector of temperatures from $T = 0^{\circ}\text{C}$ to $T = 42^{\circ}\text{C}$ with increments of 2 degrees, and display a two-column table p and T , where the first column temperatures in $^{\circ}\text{C}$, and the second column the corresponding pressures in mm Hg.

19. For many gases the temperature dependence of the heat capacity C_p of can be described in terms of a cubic equation:

$$C_p = a + bT + cT^2 + dT^3$$

The following table gives the coefficients of the cubic equation for four gases. C_p is in joules/(g mol)($^{\circ}\text{C}$) and T is in $^{\circ}\text{C}$.

Gas	a	b	c	d
SO ₂	38.91	3.904×10^{-2}	-3.105×10^{-5}	8.606×10^{-9}
SO ₃	48.50	9.188×10^{-2}	-8.540×10^{-5}	32.40×10^{-9}
O ₂	29.10	1.158×10^{-2}	-0.6076×10^{-5}	1.311×10^{-9}
N ₂	29.00	0.2199×10^{-2}	-0.5723×10^{-5}	-2.871×10^{-9}

Calculate the heat capacity for each gas at temperatures ranging between 200 and 400°C at 20°C increments. To present the results, create an 11×5 matrix where the first column is the temperature, and the second through fifth columns are the heat capacities of SO₂, SO₃, O₂, and N₂, respectively.

20. The heat capacity of an ideal mixture of four gases $C_{p_{mixture}}$ can be expressed in terms of the heat capacity of the components by the mixture equation

$$C_{p_{mixture}} = x_1 C_{p1} + x_2 C_{p2} + x_3 C_{p3} + x_4 C_{p4}$$

where $x_1, x_2, x_3,$ and x_4 are the fractions of the components, and $C_{p1}, C_{p2}, C_{p3},$ and C_{p4} are the corresponding heat capacities. A mixture of unknown quantities of the four gases $\text{SO}_2, \text{SO}_3, \text{O}_2,$ and N_2 is given. To determine the fractions of the components, the following values of the heat capacity of the mixture were measured at three temperatures:

Temperature °C	25	150	300
$C_{p_{mixture}}$ joules/(g mol)(°C)	39.82	44.72	49.10

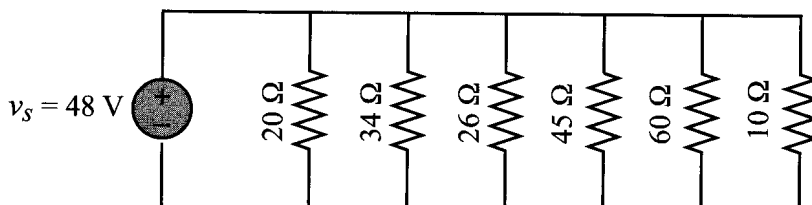
Use the equation and data in the Problem 19 to determine the heat capacity of each of the four components at the three temperatures. Then use the mixture equation to write three equations for the mixture at the three temperatures. The fourth equation is $x_1 + x_2 + x_3 + x_4 = 1$. Determine $x_1, x_2, x_3,$ and x_4 by solving the linear system of equations.

21. When several resistors are connected in an electrical circuit in parallel, the current through each of them is given by $i_n = \frac{v_s}{R_n}$ where i_n and R_n are the current through resistor n and its resistance, respectively, and v_s is the source voltage. The equivalent resistance, R_{eq} , can be determined from the equation

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

The source current is given by $i_s = v_s/R_{eq}$, and the power, P_n , dissipated in each resistor is given by $P_n = v_s i_n$.

Write a program in a script file that calculates the current through each resistor and the power dissipated in a circuit that has resistors connected in parallel. When the script file runs, it asks the user first to enter the source voltage and then to enter the resistors' resistance in a vector. The program displays a table with the resistance shown in the first column, the current through the resistor in the second column, and the power dissipated in the resistor in the third column. Following the table, the program displays the source current and the total power. Use the script file to solve the following circuit.



22. A truss is a structure made of members joined at their ends. For the truss shown in the figure, the forces in the nine members are determined by solving the following system of nine equations.

$$-\cos(45^\circ)F_1 + F_4 = 0$$

$$-F_3 - \sin(45^\circ)F_1 = 0$$

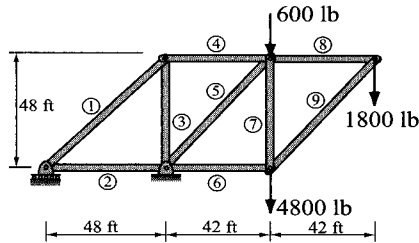
$$-F_2 + \sin(45^\circ)F_5 + F_6 = 0$$

$$-\cos(48.81^\circ)F_5 - F_4 + F_8 = 0, \quad -\sin(48.81^\circ)F_5 - F_7 = 600$$

$$-\sin(48.81^\circ)F_9 = 1800, \quad -F_8 - \cos(48.81^\circ)F_9 = 0,$$

$$F_7 + \sin(48.81^\circ)F_9 = 4800, \quad \cos(48.81^\circ)F_9 - F_6 = 0$$

Write the equations in matrix form and use MATLAB to determine the forces in the members. A positive force means tensile force and a negative force means compressive force. Display the results in a table.



23. A truss is a structure made of members joined at their ends. For the truss shown in the figure, the forces in the 11 members are determined by solving the following system of 11 equations.

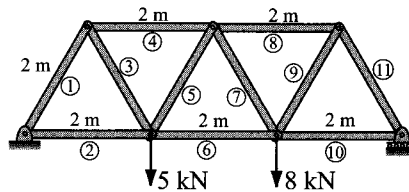
$$\frac{1}{2}F_1 + F_2 = 0, \quad \frac{\sqrt{3}}{2}F_1 = -6$$

$$-\frac{1}{2}F_1 + \frac{1}{2}F_3 + F_4 = 0, \quad -\frac{\sqrt{3}}{2}F_1 - \frac{\sqrt{3}}{2}F_3 = 0, \quad -F_2 - \frac{1}{2}F_3 + \frac{1}{2}F_5 + F_6 = 0$$

$$\frac{\sqrt{3}}{2}F_3 + \frac{\sqrt{3}}{2}F_5 = 5, \quad -F_4 - \frac{1}{2}F_5 + \frac{1}{2}F_7 + F_8 = 0, \quad -\frac{\sqrt{3}}{2}F_5 - \frac{\sqrt{3}}{2}F_7 = 0$$

$$-F_6 - \frac{1}{2}F_7 + \frac{1}{2}F_9 + F_{10} = 0, \quad \frac{\sqrt{3}}{2}F_7 + \frac{\sqrt{3}}{2}F_9 = 8, \quad -F_8 - \frac{1}{2}F_9 + \frac{1}{2}F_{11} = 0$$

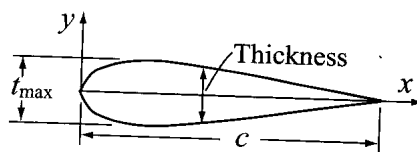
Write the equations in matrix form and use MATLAB to determine the forces in the members. A positive force means tensile force and a negative force means compressive force. Display the results in a table.



24. The graph of the function $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ passes through the points $(-4, -7.6)$, $(-2, -17.2)$, $(0.2, 9.2)$, $(1, -1.6)$, and $(4, -36.4)$. Determine the constants a , b , c , d , and e . (Write a system of five equations with five unknowns and use MATLAB to solve the equations.)

25. The surface of many airfoils can be described with an equation of the form

$$y = \mp \frac{tc}{0.2} [a_0 \sqrt{x/c} + a_1(x/c) + a_2(x/c)^2 + a_3(x/c)^3 + a_4(x/c)^4]$$



where t is the maximum thickness as a fraction of the chord length c (e.g., $t_{\max} = ct$). Given that $c = 1$ m and $t = 0.2$ m, the following values for y have been measured for a particular airfoil:

x (m)	0.15	0.35	0.5	0.7	0.85
y (m)	0.08909	0.09914	0.08823	0.06107	0.03421

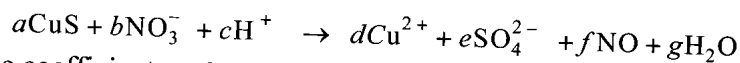
Determine the constants $a_0, a_1, a_2, a_3,$ and a_4 . (Write a system of five equations and five unknowns and use MATLAB to solve the equations.)

26. During a golf match, a certain number of points are awarded for each eagle and a different number for each birdie. No points are awarded for par, and a certain number of points are deducted for each bogey and a different number deducted for each double bogey (or worse). The newspaper report of an important match neglected to mention what these point values were, but did provide the following table of the results:

Golfer	Eagles	Birdies	Pars	Bogeys	Doubles	Points
A	1	2	10	1	1	5
B	2	3	11	0	1	12
C	1	4	10	1	10	11
D	1	3	10	12	0	8

From the information in the table write four equations in terms of four unknowns. Solve the equations for the unknown points awarded for eagles and birdies and points deducted for bogeys and double bogeys.

27. The dissolution of copper sulfide in aqueous nitric acid is described by the following chemical equation:



where the coefficients $a, b, c, d, e, f,$ and g are the numbers of the various molecule participating in the reaction and are unknown. The unknown coefficients are determined by balancing each atom on left and right and then balancing the ionic charge. The resulting equations are:

$$a = d, \quad a = e, \quad b = f, \quad 3b = 4e + f + g, \quad c = 2g, \quad -b + c = 2d - 2e$$

There are seven unknowns and only six equations. A solution can still be obtained, however, by taking advantage of the fact that all the coefficients must be positive integers. Add a seventh equation by guessing $a = 1$ and solve the system of equations. The solution is valid if all the coefficients are positive integers. If this is not the case, take $a = 2$ and repeat the solution. Continue the process until all the coefficients in the solution are positive integers.

28. The wind chill temperature, T_{wc} , is the air temperature felt on exposed skin due to wind. In U.S. customary units it is calculated by:

$$T_{wc} = 35.74 + 0.6215T - 35.75v^{0.16} + 0.4275Tv^{0.16}$$

where T is the temperature in degrees F, and v is the wind speed in mi/h. Write a MATLAB program in a script file that displays the following chart of wind chill temperature for given air temperature and wind speed in the Command Window:

	Temperature (F)								
Speed (mi/h)	40	30	20	10	0	-10	-20	-30	-40
10	34	21	9	-4	-16	-28	-41	-53	-66
20	30	17	4	-9	-22	-35	-48	-61	-74
30	28	15	1	-12	-26	-39	-53	-67	-80
40	27	13	-1	-15	-29	-43	-57	-71	-84
50	26	12	-3	-17	-31	-45	-60	-74	-88
60	25	10	-4	-19	-33	-48	-62	-76	-91

29. The stress intensity factor due to the crack shown depends upon a geometrical parameter C_I given by:

$$C_I = \sqrt{\frac{2}{\pi\alpha} \tan \frac{\pi\alpha}{2}} \left[\frac{0.923 + 0.199 \left(1 - \sin \frac{\pi\alpha}{2} \right)}{\cos \frac{\pi\alpha}{2}} \right]$$

where $\alpha = \frac{a}{b}$. Calculate C_I for α between 0.05 and 0.95 at 0.05 increments, and display the results in a two-column table with the first column showing α and the second C_I .

