15.1. IDENTIFY: \( v = f \lambda \). \( T = 1/f \) is the time for one complete vibration.

SET UP: The frequency of the note one octave higher is 1568 Hz.

EXECUTE: (a) \( \lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{784 \text{ Hz}} = 0.439 \text{ m} \). \( T = \frac{1}{f} = 1.28 \text{ ms} \).

(b) \( \lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{1568 \text{ Hz}} = 0.219 \text{ m} \).

EVALUATE: When \( f \) is doubled, \( \lambda \) is halved.

15.3. IDENTIFY: \( v = f \lambda = \lambda / T \).

SET UP: 1.0 h = 3600 s. The crest to crest distance is \( \lambda \).

EXECUTE: \( v = \frac{800 \times 10^3 \text{ m}}{3600 \text{ s}} = 220 \text{ m/s} \).

\( v = \frac{800 \text{ km}}{1.0 \text{ h}} = 800 \text{ km/h} \).

EVALUATE: Since the wave speed is very high, the wave strikes with very little warning.

15.5. IDENTIFY: \( v = f \lambda \). \( T = 1/f \).

SET UP: 1 nm = \( 10^{-9} \) m

EXECUTE: (a) \( \lambda = 400 \text{ nm} \): \( f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{400 \times 10^{-9} \text{ m}} = 7.50 \times 10^{14} \text{ Hz} \).

\( T = \frac{1}{f} = 1.33 \times 10^{-15} \text{ s} \).

\( \lambda = 700 \text{ nm} \): \( f = \frac{3.00 \times 10^8 \text{ m/s}}{700 \times 10^{-9} \text{ m}} = 4.29 \times 10^{14} \text{ Hz} \).

\( T = 2.33 \times 10^{-15} \text{ s} \). The frequencies of visible light lie between 4.29 \times 10^{14} \text{ Hz} and 7.50 \times 10^{14} \text{ Hz}.

The periods lie between 1.33 \times 10^{-15} \text{ s} and 2.33 \times 10^{-15} \text{ s}.

(b) \( T \) is very short and cannot be measured with a stopwatch.

EVALUATE: Longer wavelength corresponds to smaller frequency and larger period.

15.15. IDENTIFY and SET UP: Use Eq.(15.13) to calculate the wave speed. Then use Eq.(15.1) to calculate the wavelength.

EXECUTE: (a) The tension \( F \) in the rope is the weight of the hanging mass:

\( F = mg = (1.50 \text{ kg})(9.80 \text{ m/s}^2) = 14.7 \text{ N} \)

\( v = \sqrt{F/\mu} = \sqrt{14.7 \text{ N}/(0.0550 \text{ kg/m})} = 16.3 \text{ m/s} \)

(b) \( v = f \lambda \) so \( \lambda = v/f = (16.3 \text{ m/s})/120 \text{ Hz} = 0.136 \text{ m} \).

(c) EVALUATE: \( v = \sqrt{F/\mu} \), where \( F = mg \). Doubling m increases \( v \) by a factor of \( \sqrt{2} \). \( \lambda = \omega f \). \( f \) remains 120 Hz and \( v \) increases by a factor of \( \sqrt{2} \), so \( \lambda \) increases by a factor of \( \sqrt{2} \).

15.17. IDENTIFY: For transverse waves on a string, \( v = \sqrt{F/\mu} \). \( v = f \lambda \).

SET UP: The wire has \( \mu = m/L = (0.0165 \text{ kg})/(0.750 \text{ m}) = 0.0220 \text{ kg/m} \).

EXECUTE: (a) \( v = f \lambda = (875 \text{ Hz})(3.33 \times 10^{-2} \text{ m}) = 29.1 \text{ m/s} \). The tension is \( F = \mu v^2 = (0.0220 \text{ kg/m})(29.1 \text{ m/s})^2 = 18.6 \text{ N} \).

(b) \( v = 29.1 \text{ m/s} \)

EVALUATE: If \( \lambda \) is kept fixed, the wave speed and the frequency increase when the tension is increased.
15.7. IDENTIFY: Use Eq.(15.1) to calculate $v$. $T = 1/f$ and $k$ is defined by Eq.(15.5). The general form of the wave function is given by Eq.(15.8), which is the equation for the transverse displacement.

SET UP: $v = 8.00 \text{ m/s}$, $A = 0.0700 \text{ m}$, $\lambda = 0.320 \text{ m}$

EXECUTE: (a) $v = f \lambda$ so $f = v/\lambda = (8.00 \text{ m/s})/(0.320 \text{ m}) = 25.0 \text{ Hz}$

$T = 1/f = 1/25.0 \text{ Hz} = 0.0400 \text{ s}$

$k = 2\pi/\lambda = 2\pi \text{ rad/0.320 m} = 19.6 \text{ rad/m}$

(b) For a wave traveling in the $-x$-direction,

$y(x, t) = A \cos 2\pi(x/\lambda + t/T)$ (Eq.(15.8)).

At $x = 0$, $y(0, t) = A \cos 2\pi(t/T)$, so $y = A$ at $t = 0$. This equation describes the wave specified in the problem.

Substitute in numerical values:

$y(x, t) = (0.0700 \text{ m}) \cos(2\pi(x/0.320 \text{ m} + t/0.0400 \text{ s}))$

Or, $y(x, t) = (0.0700 \text{ m}) \cos((19.6 \text{ rad/m})x + (157 \text{ rad/s})t)$.

(c) From part (b), $y = (0.0700 \text{ m}) \cos(2\pi(x/0.320 \text{ m} + t/0.0400 \text{ s}))$.

Plug in $x = 0.360 \text{ m}$ and $t = 0.150 \text{ s}$:

$y = (0.0700 \text{ m}) \cos(2\pi(0.360 \text{ m}/0.320 \text{ m} + 0.150 \text{ s}/0.0400 \text{ s}))$

$y = (0.0700 \text{ m}) \cos(2\pi(4.875 \text{ rad})) = +0.0495 \text{ m} = +4.95 \text{ cm}$

(d) In part (c) $t = 0.150 \text{ s}$.

$y = A$ means $\cos(2\pi(x/\lambda + t/T)) = 1$

$\cos \theta = 1$ for $\theta = 0, 2\pi, 4\pi, ..., n(2\pi)$ or $n = 0, 1, 2, ...$

So $y = A$ when $2\pi(x/\lambda + t/T) = n(2\pi)$ or $x/\lambda + t/T = n$

$t = T(n - x/\lambda) = (0.0400 \text{ s})(n - 0.360 \text{ m}/0.320 \text{ m}) = (0.0400 \text{ s})(n - 1.125)$

For $n = 4$, $t = 0.1150 \text{ s}$ (before the instant in part (c))

For $n = 5$, $t = 0.1550 \text{ s}$ (the first occurrence of $y = A$ after the instant in part (c)) Thus the elapsed time is $0.1550 \text{ s} - 0.1150 \text{ s} = 0.0400 \text{ s}$.

EVALUATE: Part (d) says $y = A$ at $0.115 \text{ s}$ and next at $0.155 \text{ s}$; the difference between these two times is $0.040 \text{ s}$, which is the period. At $t = 0.150 \text{ s}$ the particle at $x = 0.360 \text{ m}$ is at $y = 4.95 \text{ cm}$ and traveling upward. It takes $T/4 = 0.0100 \text{ s}$ for it to travel from $y = 0$ to $y = A$, so our answer of $0.0050 \text{ s}$ is reasonable.

15.8. IDENTIFY: The general form of the wave function for a wave traveling in the $-x$-direction is given by Eq.(15.8). The time for one complete cycle to pass a point is the period $T$ and the number that pass per second is the frequency $f$. The speed of a crest is the wave speed $v$ and the maximum speed of a particle in the medium is $v_{\text{max}} = \omega A$.

SET UP: Comparison to Eq.(15.8) gives $A = 3.75 \text{ cm}$, $k = 0.450 \text{ rad/cm}$ and $\omega = 5.40 \text{ rad/s}$.

EXECUTE: (a) $T = \frac{2\pi \text{ rad}}{\omega} = \frac{2\pi \text{ rad}}{5.40 \text{ rad/s}} = 1.16 \text{ s}$. In one cycle a wave crest travels a distance

$\lambda = \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{0.450 \text{ rad/cm}} = 1.40 \text{ m}$.

(b) $k = 0.450 \text{ rad/cm}$, $f = 1/T = 0.862 \text{ Hz} = 0.862 \text{ waves/second}$.

(c) $v = \lambda f = (3.862 \text{ Hz})(0.140 \text{ m}) = 0.121 \text{ m/s}$, $v_{\text{max}} = \omega A = (5.40 \text{ rad/s})(3.75 \text{ cm}) = 0.202 \text{ m/s}$.

EVALUATE: The transverse velocity of the particles in the medium (water) is not the same as the velocity of the wave.

15.20. IDENTIFY: Apply Eq.(15.25).

SET UP: $\omega = 2\pi f$, $\mu = m/L$.

EXECUTE: (a) $P_w = \frac{1}{2} \sqrt{\mu F \omega^2 A^2}$, $P_w = \frac{1}{2} \sqrt{\left(3.00 \times 10^{-3} \text{ kg} \right) \left(25.0 \text{ N} \right) \left(2\pi \left(120.0 \text{ Hz} \right) \right)^2 \left(1.6 \times 10^{-2} \text{ m} \right)^2} = 0.223 \text{ W}$

or $0.22 \text{ W}$ to two figures.

(b) $P_w$ is proportional to $A^2$, so halving the amplitude quarters the average power, to $0.056 \text{ W}$.

EVALUATE: The average power is also proportional to the square of the frequency.
15.23. **IDENTIFY and SET UP:** Apply Eq.(15.26) to relate \( I \) and \( r \).

Power is related to intensity at a distance \( r \) by \( P = I(4\pi r^2) \). Energy is power times time.

**EXECUTE:**  
(a) \( I_1 r_1^2 = I_2 r_2^2 \)

\( I_2 = I_1 (r_1 / r_2)^2 = (0.026 \text{ W/m}^2)(4.3 \text{ m}/3.1 \text{ m})^2 = 0.050 \text{ W/m}^2 \)

(b) \( P = 4\pi r^2 I = 4\pi (4.3 \text{ m})^2 (0.026 \text{ W/m}^2) = 6.04 \text{ W} \)

Energy = \( P t = (6.04 \text{ W})(3600 \text{ s}) = 2.2\times10^4 \text{ J} \)

**EVALUATE:** We could have used \( r = 3.1 \text{ m} \) and \( I = 0.050 \text{ W/m}^2 \) in \( P = 4\pi r^2 I \) and would have obtained the same \( P \). Intensity becomes less as \( r \) increases because the radiated power spreads over a sphere of larger area.

15.30. **IDENTIFY:** Apply the principle of superposition.

**SET UP:** The net displacement is the algebraic sum of the displacements due to each pulse.

**EXECUTE:** The shape of the string at each specified time is shown in Figure 15.30.

**EVALUATE:** The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

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**Figure 15.30**
15.31. **IDENTIFY:** Apply the principle of superposition.

**SET UP:** The net displacement is the algebraic sum of the displacements due to each pulse.

**EXECUTE:** The shape of the string at each specified time is shown in Figure 15.31.

**EVALUATE:** The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

\[ \begin{align*}
4.00 \text{ s} & \quad \text{\square} \\
6.00 \text{ s} & \quad \text{\square} \\
10.0 \text{ s} & \quad \text{\square}
\end{align*} \]

Figure 15.31

15.33. **IDENTIFY and SET UP:** Nodes occur where \( \sin kx = 0 \) and antinodes are where \( \sin kx = \pm 1 \).

**EXECUTE:** Eq. (15.28): \( y = (A_{yw} \sin kx) \sin \alpha t \)

(a) At a node \( y = 0 \) for all \( t \). This requires that \( \sin kx = 0 \) and this occurs for \( kx = n\pi, \ n = 0, 1, 2, \ldots \)

\[ x = n\pi k = \frac{n\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})n, \ n = 0, 1, 2, \ldots \]

(b) At an antinode \( \sin kx = \pm 1 \) so \( y \) will have maximum amplitude. This occurs when \( kx = (n + \frac{1}{2})\pi, \ n = 0, 1, 2, \ldots \)

\[ x = (n + \frac{1}{2})\pi k = \frac{(n + \frac{1}{2})\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})(n + \frac{1}{2}), \ n = 0, 1, 2, \ldots \]

**EVALUATE:** \( \lambda = 2\pi/k = 2.66 \text{ m} \). Adjacent nodes are separated by \( \lambda/2 \), adjacent antinodes are separated by \( \lambda/2 \), and the node to antinode distance is \( \lambda/4 \).

15.39. **IDENTIFY:** Use Eq. (15.1) for \( v \) and Eq. (15.13) for the tension \( F \). \( v_y = \partial y/\partial t \) and \( a_y = \partial v_y/\partial t \).

(a) **SET UP:** The fundamental standing wave is sketched in Figure 15.39.

\[ \begin{align*}
L &= 0.800 \text{ m} \\
f &= 60.0 \text{ Hz}
\end{align*} \]

From the sketch, \( \lambda/2 = L \) so \( \lambda = 2L = 1.60 \text{ m} \)

**EXECUTE:** \( v = f \lambda = (60.0 \text{ Hz})(1.60 \text{ m}) = 96.0 \text{ m/s} \)

(b) The tension is related to the wave speed by Eq. (15.13):

\[ \begin{align*}
v &= \sqrt{F/\mu} \quad \text{so} \quad F = \mu v^2 \\
\mu &= m/L = 0.0400 \text{ kg}/0.800 \text{ m} = 0.0500 \text{ kg/m} \\
F &= \mu v^2 = (0.0500 \text{ kg/m})(96.0 \text{ m/s})^2 = 461 \text{ N.}
\end{align*} \]

(e) \( \omega = 2\pi f = 377 \text{ rad/s} \) and \( y(x, t) = A_{yw} \sin kx \sin \alpha t \)

\[ \begin{align*}
v_y &= \omega A_{yw} \sin kx \cos \alpha t; \quad a_y = -\omega^2 A_{yw} \sin kx \sin \alpha t \\
(v_y)_{\text{max}} &= \omega A_{yw} = (377 \text{ rad/s})(0.300 \text{ cm}) = 1.13 \text{ m/s.} \\
(a_y)_{\text{max}} &= \omega^2 A_{yw} = (377 \text{ rad/s})^2 (0.300 \text{ cm}) = 426 \text{ m/s}^2.
\end{align*} \]

**EVALUATE:** The transverse velocity is different from the wave velocity. The wave velocity and tension are similar in magnitude to the values in the Examples in the text. Note that the transverse acceleration is quite large.
15.47. **IDENTIFY:** For the fundamental, \( f_i = \frac{v}{2L} \). \( v = \sqrt{F/\mu} \). A standing wave on a string with frequency \( f \) produces a sound wave that also has frequency \( f \).

**SET UP:** \( f_i = 245 \text{ Hz} \). \( L = 0.635 \text{ m} \).

**EXECUTE:** 
(a) \( v = 2f/L = 2(245 \text{ Hz})(0.635 \text{ m}) = 311 \text{ m/s} \).

(b) The frequency of the fundamental mode is proportional to the speed and hence to the square root of the tension; 
(245 Hz)\sqrt{1.01} = 246 \text{ Hz} \).

(c) The frequency will be the same, 245 Hz. The wavelength will be \( \lambda_{\text{sw}} = \frac{v_{\text{sw}}}{f} = \frac{344 \text{ m/s}}{(245 \text{ Hz})} = 1.40 \text{ m} \), which is larger than the wavelength of standing wave on the string by a factor of the ratio of the speeds.

**EVALUATE:** Increasing the tension increases the wave speed and this in turn increases the frequencies of the standing waves. The wavelength of each normal mode depends only on the length of the string and doesn't change when the tension changes.